Challenges in Computational Astrophysics with Black Holes

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NUMERICAL AND ANALYTICAL RELATIVITY (NAR-2024)
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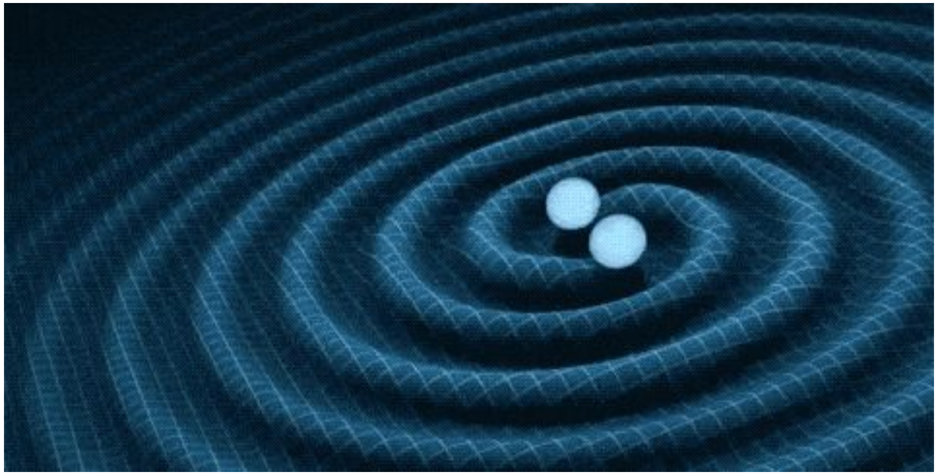
Black Hole *Binaries* emit Gravitational Waves

- Orbiting systems of stars evolve into binary black holes. They emit gravitational waves and lose orbital energy.
- Orbits keeps tightening till the black holes collide. Remnant is also a black hole.
- Remnant black hole is very distorted at birth. It emits gravitational waves and settles down to a quiescent state.



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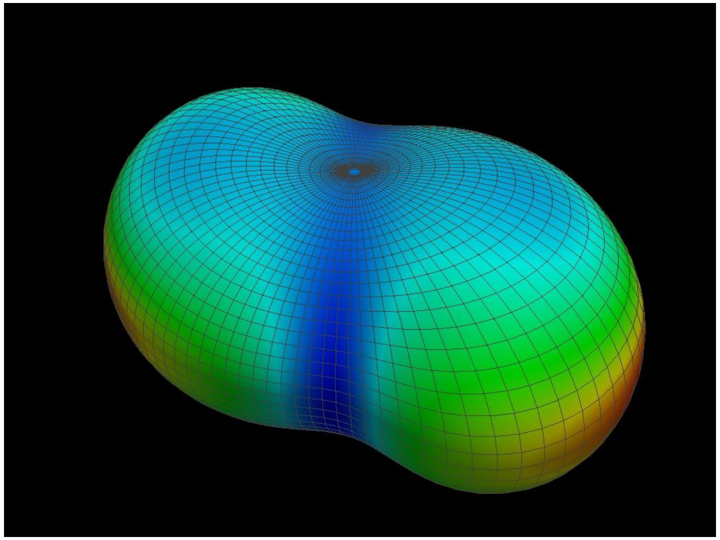

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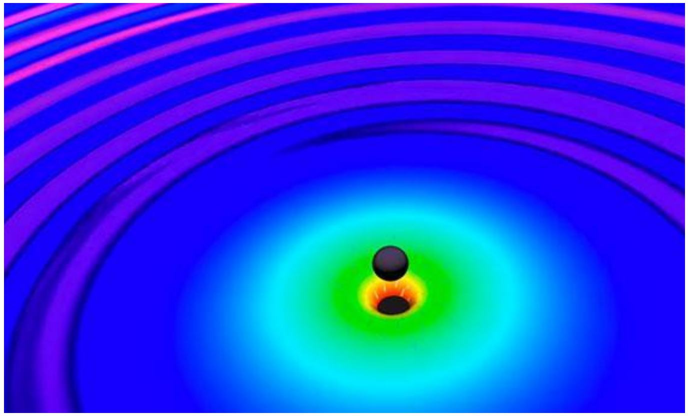
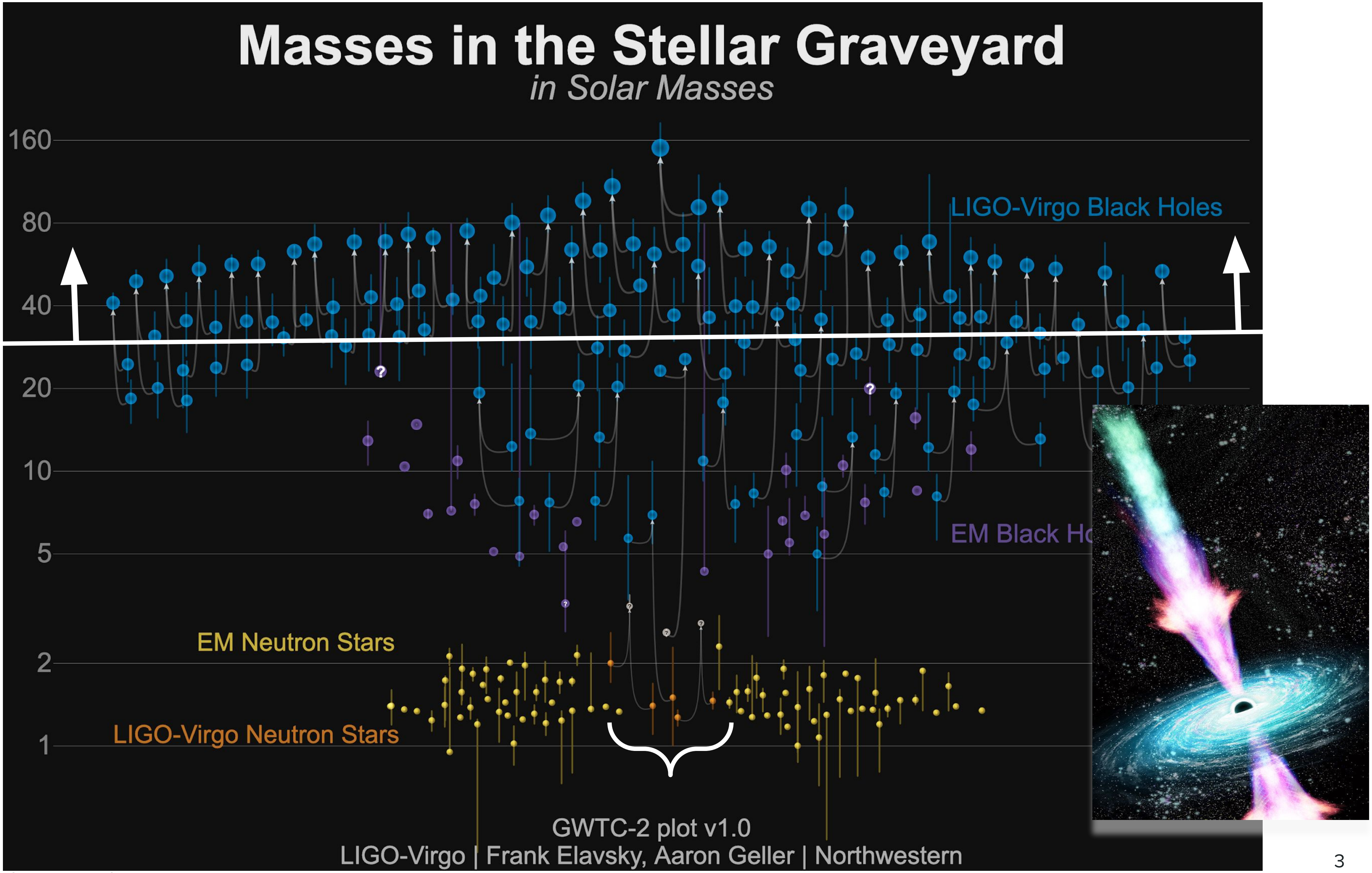


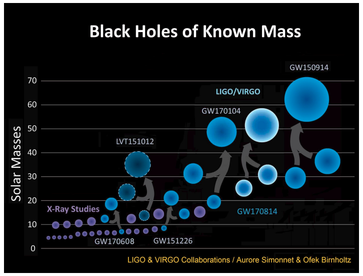
Image credit: Kavli Foundation, LSC; https://cqgplus.files.wordpress.com;

GW Observations with LIGO

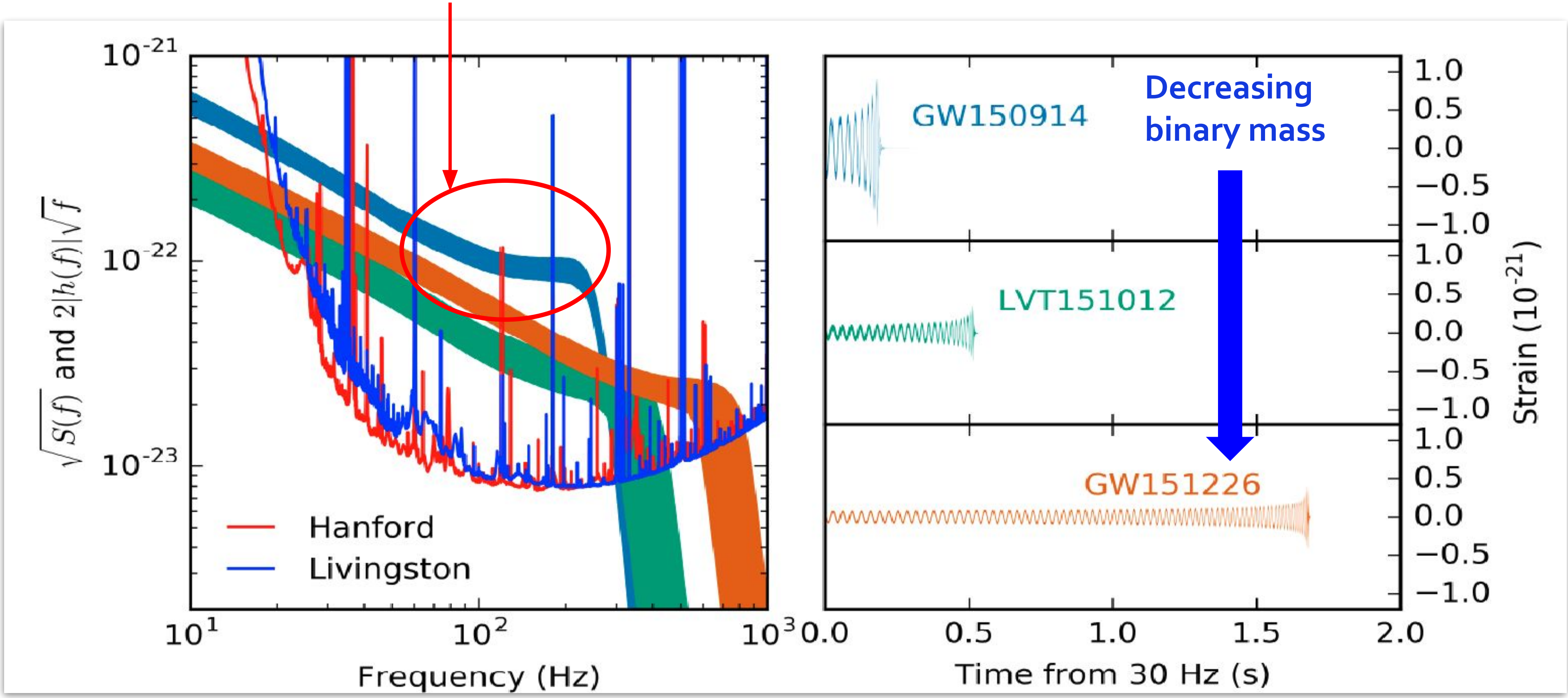


PC: University of Bath

GW observations: these black holes are heavy!

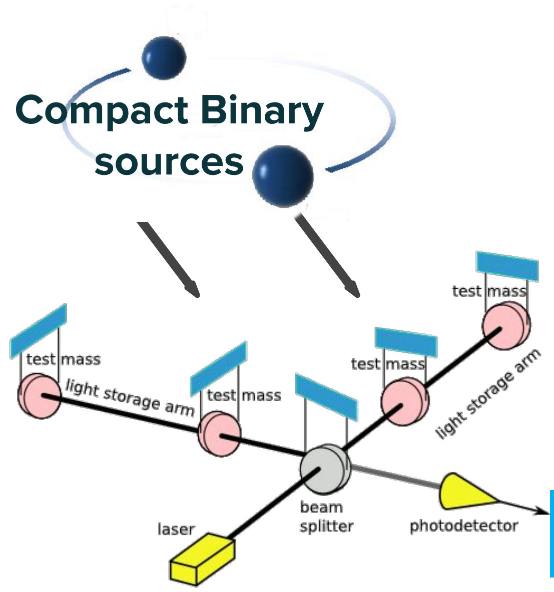


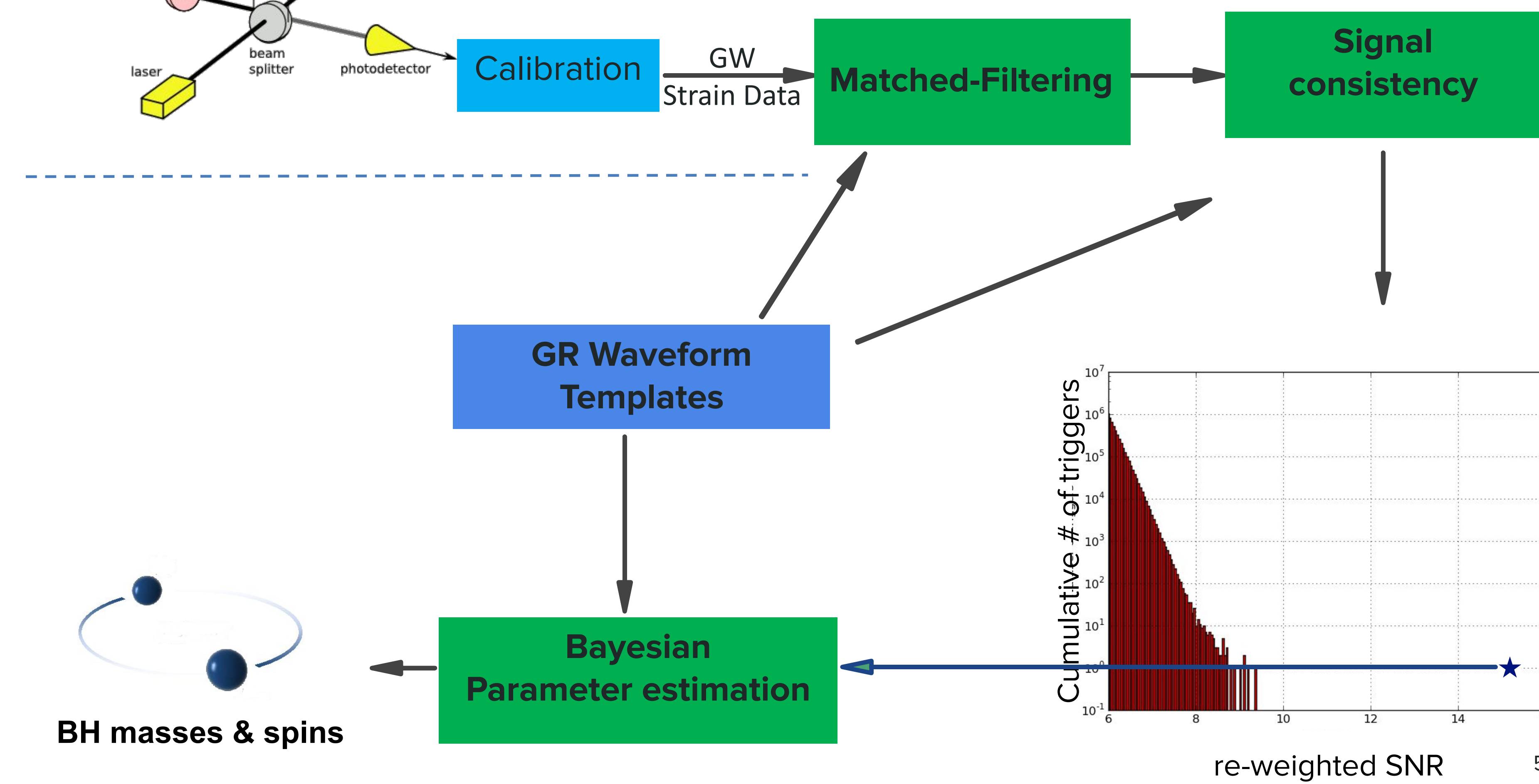
Massive binaries → Strong-field non-linear GR dynamics observable!



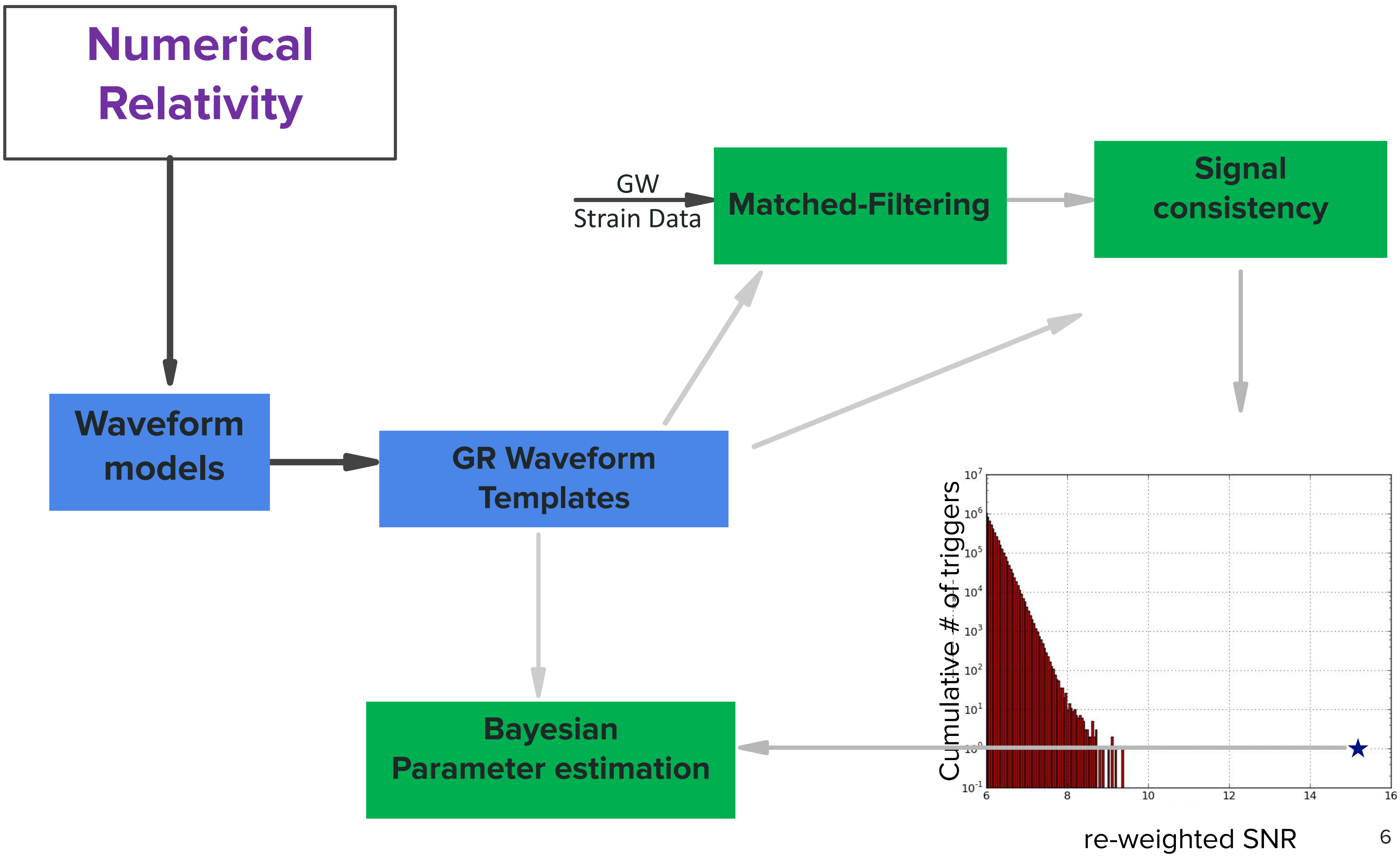
Role of Numerical Relativity

Anatomy of GW astronomy





Role of Numerical Relativity

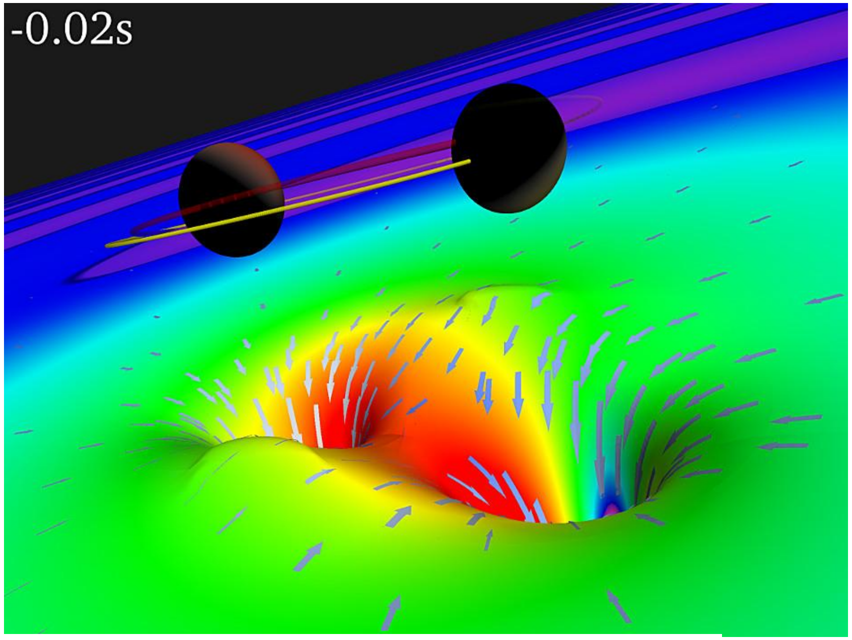


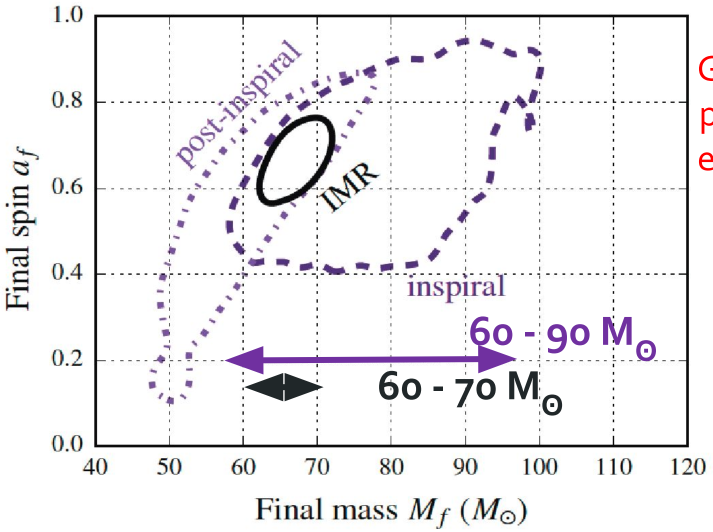
Numerical simulations are *necessary* for BBH science

For BBH, last ~10 orbits, merger and ringdown, can only be computed with full numerical solutions of Einstein's equations.

Without Numerical Relativity:

- GW events like GW150914,GW151226, GW170104 - would have had much lower significance ("probable" vs "confident" detection)
- If GW150914's source merged 25% further away, it would not even have been detected in Livingston
- We would only very approximately determine black hole characteristics from the GW signal
- We could not have tested GR



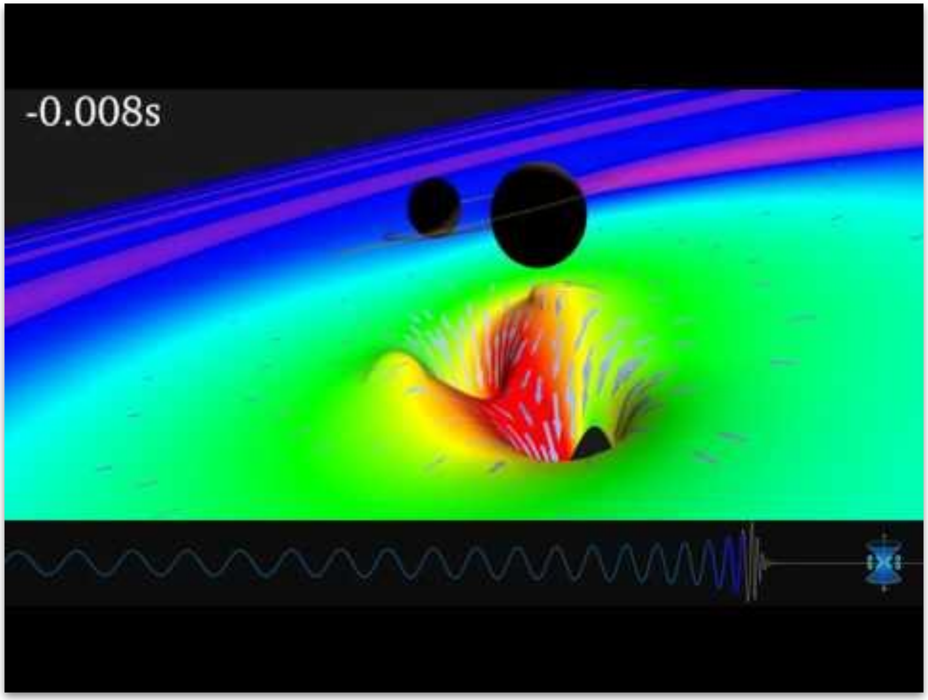


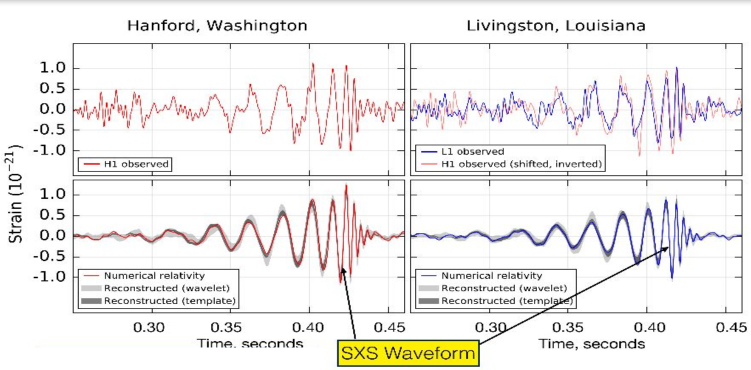
GW150914 parameter estimation

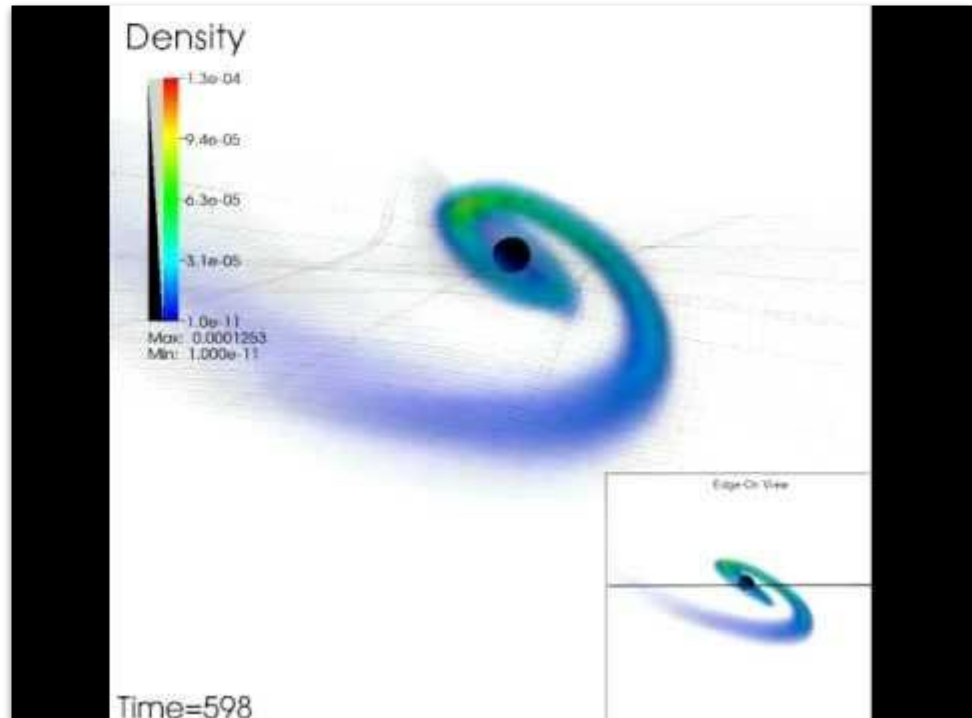
Image credit: Harald Pfeiffer, SXS Collaboration;
Abbott ..PK..et al (2016), Phys. Rev. Lett. **116**, 061102;

Simulating Binary Black Hole Coalescence

Black holes and Neutron stars







GR and Einstein's Equations

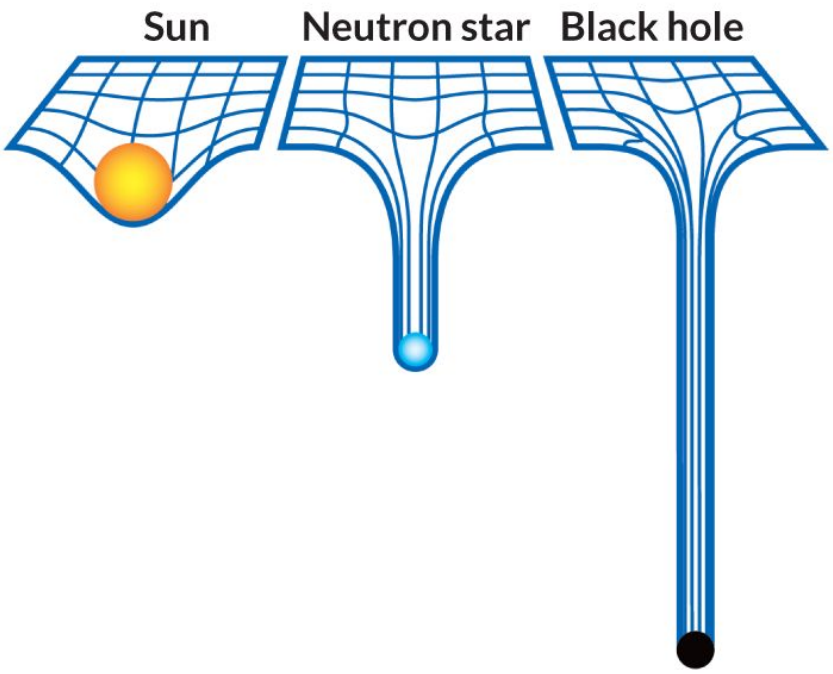
- Newtonian gravity: $\vec{\nabla}^2\Phi = 4\pi G\rho$ $\quad \vec{a} = -\vec{\nabla}\Phi$

  Flat Space-time

- Einsteinian gravity:

  (i) Curved space-time

  (ii) Geometry represented by the space-time metric $g_{ab}(\vec{x}, t)$, a,b = { x,y,z,t }.
  Metric is determined by solving Einstein Field Equations



GR and Einstein's Equations

- (ii) Geometry represented by the space-time metric $g_{ab}(\vec{x}, t)$, a,b = { x,y,z,t }. Metric is determined by solving Einstein Field Equations:

$$R_{ab}[g_{ab}(\vec{x}, t)] = 0, \qquad a, b = 0, \ldots 3$$

$$R_{ab} = \sum_{d=0}^{3} \partial_d \Gamma^d_{ab} - \sum_{d=0}^{3} \partial_b \Gamma^d_{da} + \sum_{c,d=0}^{3} \Gamma^c_{cd} \Gamma^d_{ab} - \sum_{c,d=0}^{3} \Gamma^d_{bc} \Gamma^c_{da}$$

$$\Gamma^a_{bc} = \sum_{d=1}^{4} (g_{ad})^{-1} \left(\partial_b g_{db} + \partial_c g_{bd} - \partial_d g_{bc} \right)$$

ANNALS OF PHYSICS: **29**, 304–331 (1964)

The Two-Body Problem in Geometrodynamics

SUSAN G. HAHN

International Business Machines Corporation, New York, New York

AND

RICHARD W. LINDQUIST

100 kFlops*

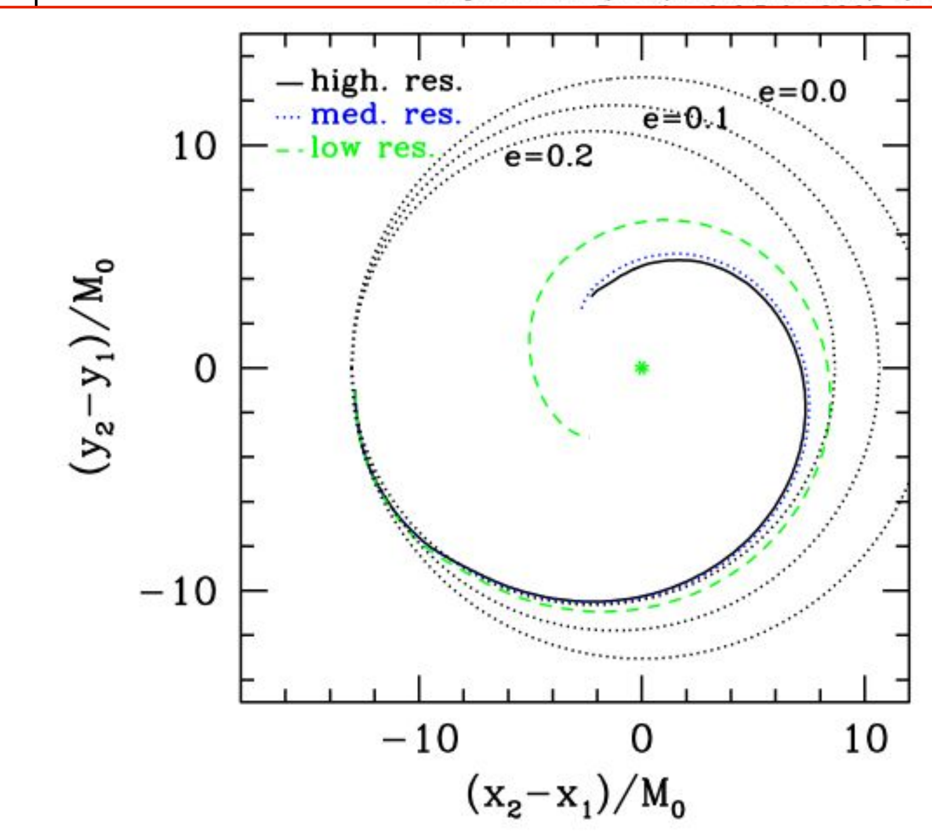
The numerical calculations were carried out on an IBM 7090 electronic computer. The parameters a and μ_0 were both set equal to unity; the mesh lengths were assigned the values $h_1 = 0.02$, $h_2 = \pi/150 \approx 0.021$, yielding a 51 × 151 mesh. The calculations of all unknown functions, including a great number of input-output operations and some built-in checking procedures, took approximately four minutes per time step. Different check routines indicated that results close to the point $\mu = 0$, $\eta = 0$ lost accuracy fairly quickly. Since these would, in the long run, influence meshpoints further away, the computations were stopped after the 50th time step, when the total time elapsed was approximately 1.8. Some of the results are shown in Table I.

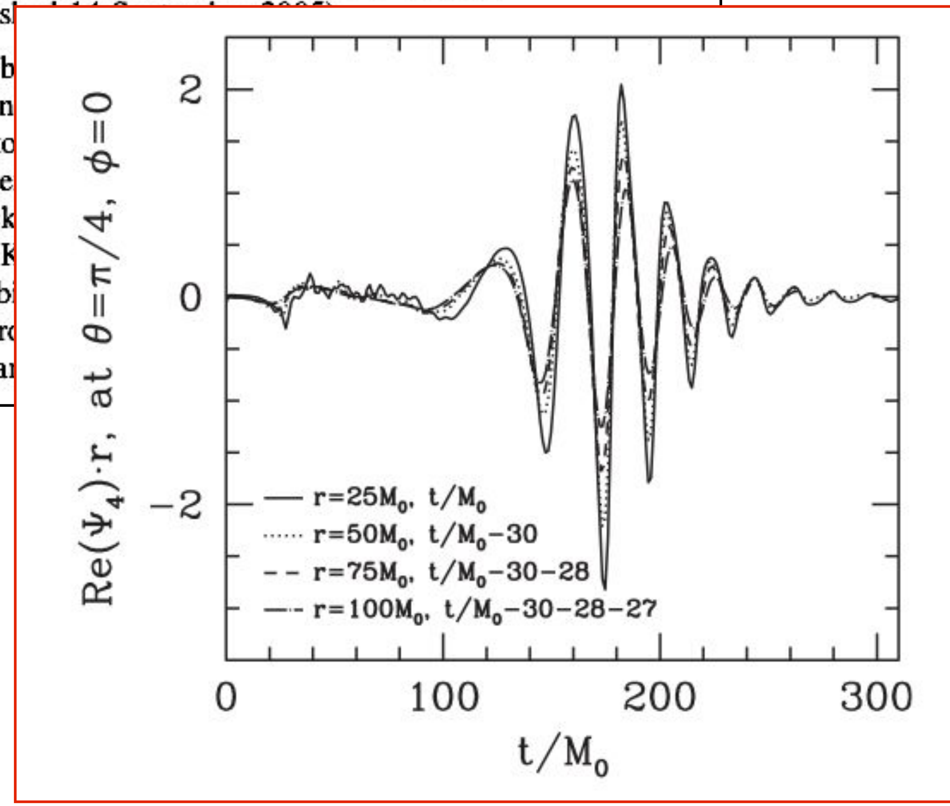
* https://en.wikipedia.org/wiki/IBM_7090

PRL **95,** 121101 (2005) PHYSICAL REVIEW LETTERS week e
16 SEPTEM

Evolution of Binary Black-Hole Spacetimes

Frans Pretorius[1,2,*]
[1]*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA*
[2]*Department of Physics, University of Alberta, Edmonton, AB T6G 2J1 Canada*





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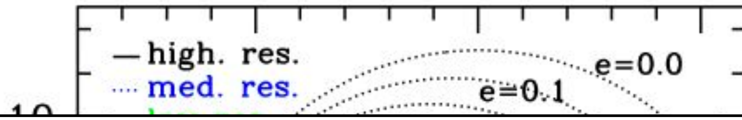

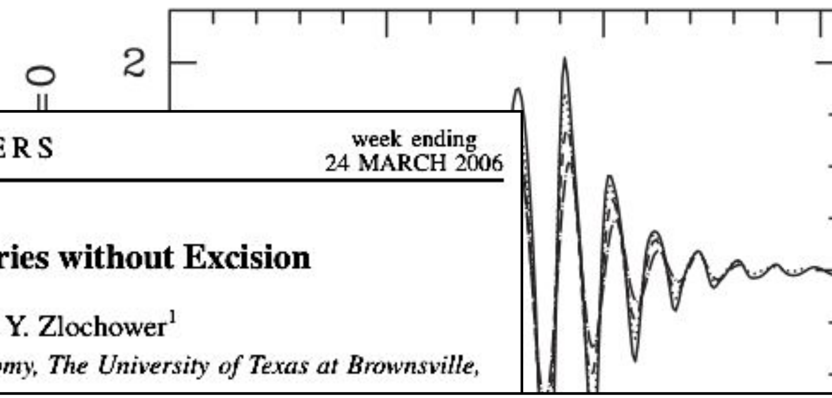

PRL **96,** 111101 (2006) PHYSICAL REVIEW LETTERS week ending
24 MARCH 2006

Accurate Evolutions of Orbiting Black-Hole Binaries without Excision

M. Campanelli,[1] C. O. Lousto,[1] P. Marronetti,[2] and Y. Zlochower[1]
[1]*Department of Physics and Astronomy, and Center for Gravitational Wave Astronomy, The University of Texas at Brownsville,*
[2]*Depart*

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PRL **96,** 111102 (2006) PHYSICAL REVIEW LETTERS week ending
24 MARCH 2006

Gravitational-Wave Extraction from an Inspiraling Configuration of Merging Black Holes

John G. Baker,[1] Joan Centrella,[1] Dae-Il Choi,[1,2] Michael Koppitz,[1] and James van Meter[1]
[1]*Gravitational Astrophysics Laboratory, NASA Goddard Space Flight Center, 8800 Greenbelt Road, Greenbelt, Maryland 20771, USA*
[2]*Universities Space Research Association, 10211 Wincopin Circle, Suite 500, Columbia, Maryland 21044, USA*
(Received 15 November 2005; published 22 March 2006)

We present new ideas for evolving black holes through a computational grid without excision, which enable accurate and stable evolutions of binary black hole systems with the accurate determination of gravitational waveforms directly from the wave zone region. Rather than excising the black hole interiors, our approach follows the "puncture" treatment of black holes, but utilizing a new gauge condition which allows the black holes to move successfully through the computational domain. We apply these techniques to an inspiraling binary, modeling the radiation generated during the final plunge and ringdown. We demonstrate convergence of the waveforms and good conservation of mass-energy, with just over 3% of the system's mass converted to gravitational radiation.

Solving Einstein Equations: 3+1 split

- Goal: Space-time metric g_{ab} satisfying

$$R_{ab}[g_{ab}] = 0$$

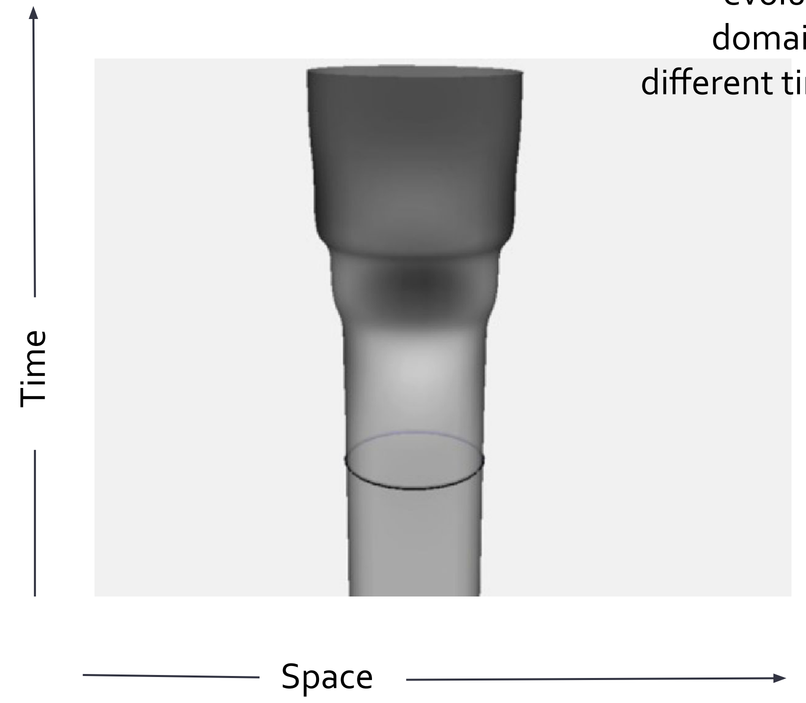
- Split space-time into space *and* time

Evolution equations

$$\partial_t g_{ij} = \dots$$
$$\partial_t K_{ij} = \dots$$

Constraints

$$R[g_{ij}] + K^2 - K_{ij}K^{ij} = 0$$
$$\nabla_j \left(K^{ij} - g^{ij}K\right) = 0$$



Snapshots of evolution domain at different times

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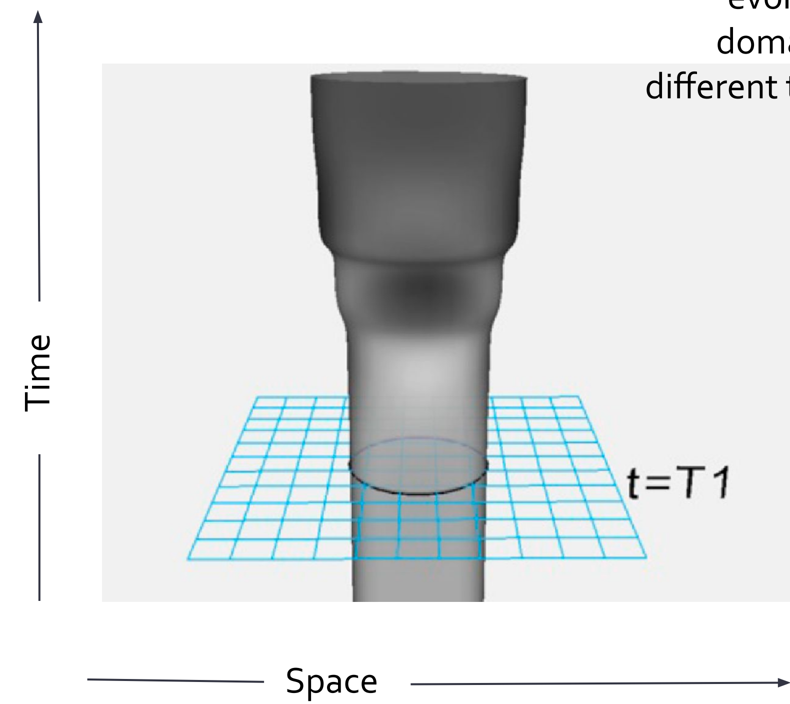

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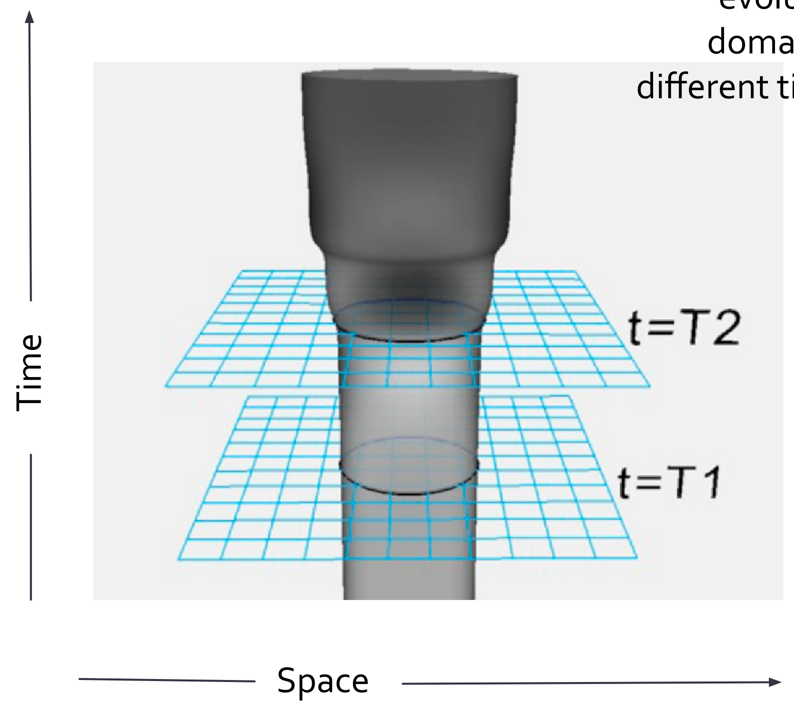

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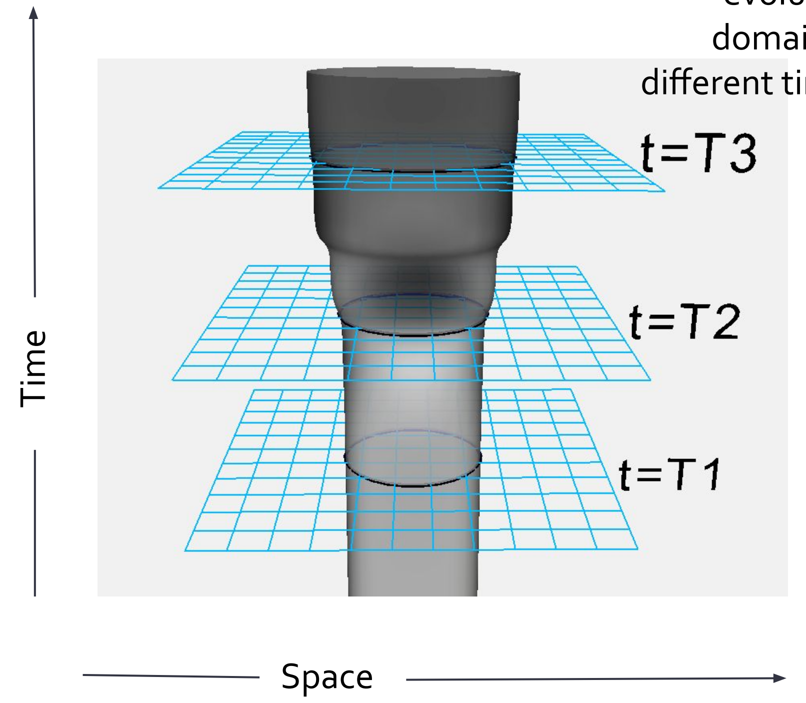


Maxwell's equations

$$\partial_t \vec{E} = \nabla \times \vec{B}$$
$$\partial_t \vec{B} = -\nabla \times \vec{E}$$
$$\nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \vec{B} = 0$$

What makes it challenging:
Multiple length/time scales, Courant limit, Accuracy required

1. Multiple length scales:

- Size of BH ~ **O(1M)**
- Separation ~ **O(10M)**
- Wavelength λ_{GW} ~ **O(100M)**
- Wave extraction ~ **several λ_{GW}**
- GW flux, that drives the inspiral, is small:

$$\dot{E}/E \sim 10^{-5}$$

What makes it challenging:
Multiple length/time scales, Courant limit, Accuracy required

1. **Multiple length/time scales** (**BH size** ~O(1); λ_{GW} ~O(100), **Outer bdry** ~O(1000))

2. **Which coordinates to use** (for a spacetime one doesn't know yet)?

3. **Putting Black holes (singularity) on a grid**

4. **Einstein constraints grew exponentially:** for many ~~years~~ decades

5. **Resolving shocks (discontinuities)**

6. **Computational Challenges:**

- 20–50 variables
- Global timestep too small
- Computing efficiency

7. **High accuracy required by LIGO:**

- Absolute phase error << 1 rad / 20+ orbits

What makes it challenging:

Multiple length/time scales, Courant limit, Accuracy required

1. Multiple length scales

2. Which coordinates to use (for a spacetime one doesn't know yet)?

3. Putting Black holes (singularity) on a grid

4. Einstein constraints C = 0: for many years, $\partial_t C \sim C$

5. Resolving shocks (discontinuities)

6. Computational Challenges

7. High accuracy required by LIGO

But, in vacuum, solutions are smooth $\Rightarrow$ **Spectral methods**

Spectral Einstein Code (SpEC*)

Goal: Solve Einstein's equations to enable robust gravitational-wave science

In development since 2002

650,000 lines, 130 publications

Simulations of Extreme Spacetimes (SXS) collaboration

Brief timeline of developments:

2005, Pretorius: First BBH merger

2006, Goddard group & UBT group: BBH mergers with different formulation

2007, BBH mergers with SpEC code: Now leading code to provide waveforms for LIGO

* http://www.black-holes.org/

SpEC: (non-local) Spectral discretization

Evolution quantities are smoothly varying.

- Expand them in basis-functions, solve for coefficients

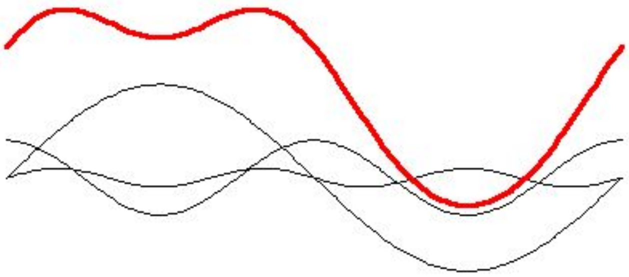
$$u(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi_k(x)$$

- Compute derivatives *exactly*

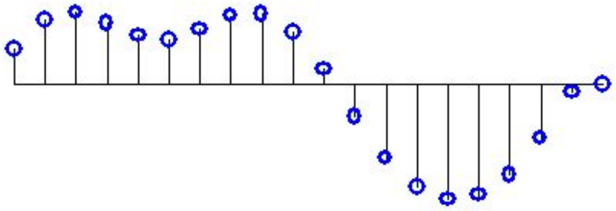
$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi'_k(x)$$

- Compute nonlinearities in physical space

Spectral



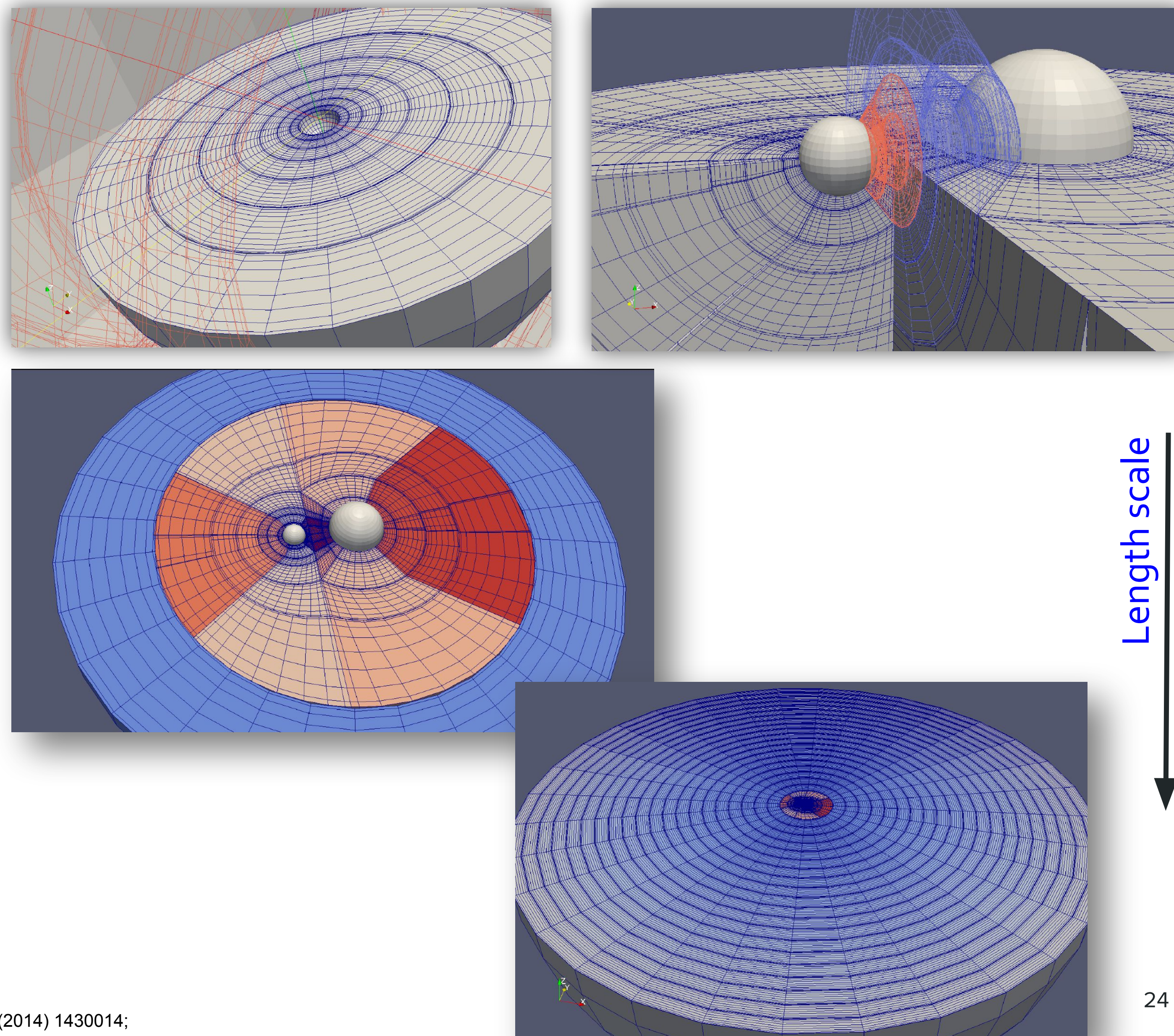
Finite differences



Domain decomposition

I1: Chebyshev polynomials
S1: Fourier
S2: Scalar Ylm
B2: One-sided Jacobi polynomials.

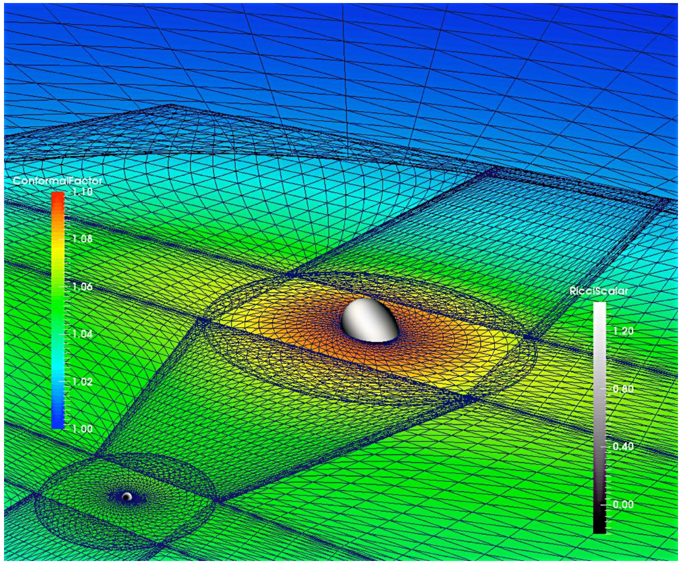
- Local resolution controllable dynamically
- **MPI parallelization by sub-domain**

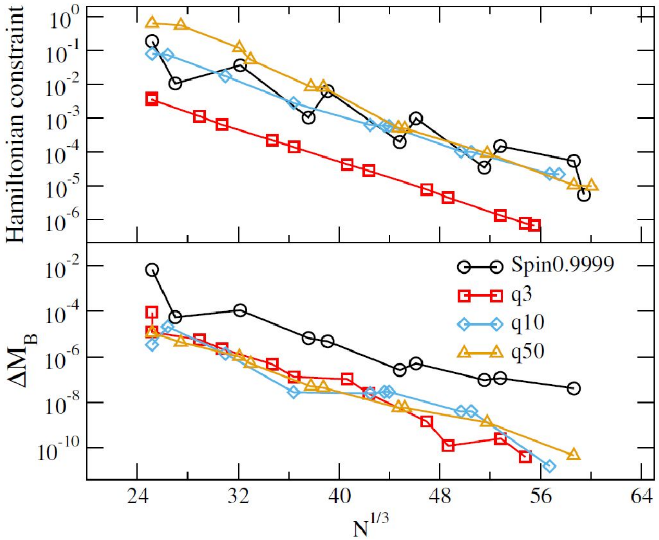


Bela Szilagyi, Int. J Mod. Phys. D, Vol. 23, No. 6 (2014) 1430014;

Initial data: Solve Einstein constraint equations

- Need {K_{ij}, g_{ij}} that satisfy Einstein constraints
- Conformal formulation of constraints. Free data provided for {conformal 3-metric, K, and their ∂_t}
- Solve constraints for u^{δ} = {ψ, N, N^k}. Boundary conditions for u^{δ}, on S^A & S^B & ∂D, give desired physics: BH spins and orbital properties.
- Second-order coupled Elliptic PDEs





Pfeiffer et al, Comput.Phys.Commun. 152 (2003) 253-273;
[1] Ossokine et al, Class. Quant Grav, Vol 32, 24 (2015)

Initial data: Solve Einstein constraint equations

Newton Raphson for elliptic equations

- Need $\{K_{ij}, g_{ij}\}$ that satisfy Einstein constraints
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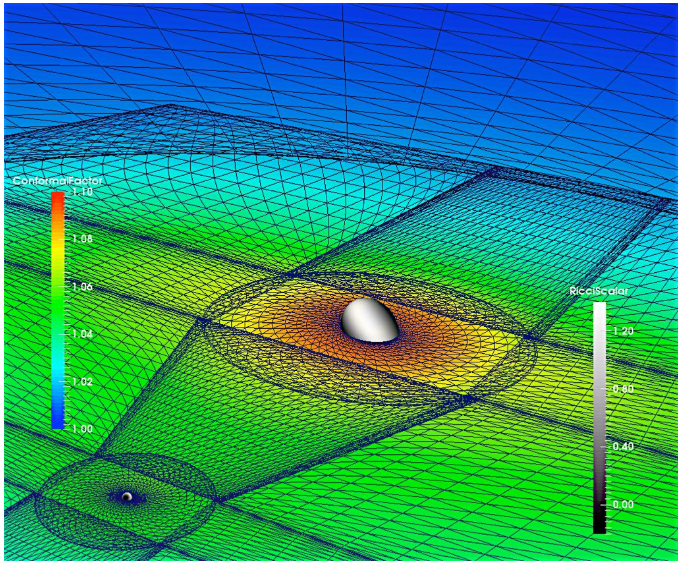
- Second-order coupled Elliptic PDEs :

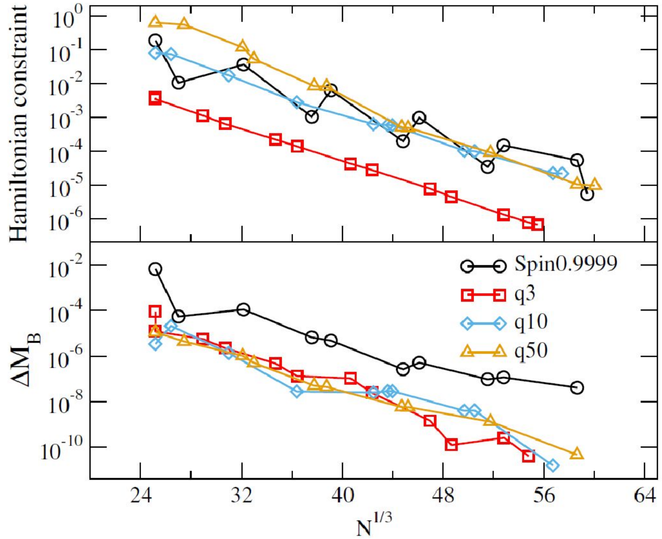
$$\mathcal{S}[\underline{u}(\vec{x}))] = 0$$

- Expand on spectral bases in each sub-domain:

$$\underline{u}(\vec{x}) = \sum_i \tilde{u}_i \Phi_i(\vec{x})$$

- Linearize ***S*** and solve with Newton-Raphson
- Adaptive refinement of grid for high mass-ratios





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Evolution: First-order system

We evolve a first order representation of Einstein evolution equations:

$$\partial_t u^\alpha + A^{k\,\alpha}_{\beta}\,\partial_k u^\beta = F^\alpha$$

$$u^\alpha = \{g_{ab}, \Pi_{ab} = -t^c\partial_c g_{ab}, \Phi_{iab} = \partial_i g_{ab}\}$$

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Principal parts:

$$\partial_t g_{ab} - N^k \partial_k g_{ab} \simeq 0,$$
$$\partial_t \Pi_{ab} - N^k \partial_k \Pi_{ab} + N \psi^{ki} \partial_k \Phi_{iab} \simeq 0,$$
$$\partial_t \Phi_{iab} - N^k \partial_k \Phi_{iab} + N \partial_i \Pi_{ab} \simeq 0.$$

Evolution: First-order system

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Principal parts:

$$\begin{aligned} \partial_t g_{ab} - N^k\partial_k g_{ab} &\simeq 0, \\ \partial_t \Pi_{ab} - N^k\partial_k \Pi_{ab} + N\psi^{ki}\partial_k\Phi_{iab} &\simeq 0, \\ \partial_t \Phi_{iab} - N^k\partial_k\Phi_{iab} + N\partial_i\Pi_{ab} &\simeq 0. \end{aligned}$$

Subject to constraints:

$$C_a = C_{iab} = 0$$

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$$\partial_t \Phi_{iab} - N^k\partial_k \Phi_{iab} + N\partial_i \Pi_{ab} \simeq 0.$$

Subject to constraints:

$$C_a = C_{iab} = 0$$

… which can grow exponentially!

$$\partial_t C \propto C$$

Constraint Damping: Example

An illustrative example :
scalar wave in flat spacetime

$$\partial_\mu \partial^\mu \psi = 0$$

First-order form:

$$\begin{cases} \partial_t \psi + \Pi = 0, \\ \partial_t \Pi + \partial^i \Phi_i = 0, \\ \partial_t \Phi_i + \partial_i \Pi = 0. \end{cases}$$

Constraint:

$$C_i = \partial_i \psi - \Phi_i = 0$$

Constraint evolution:

$$\partial_t C_i = 0$$

Constraint Damping: Example

An illustrative example :
scalar wave in flat spacetime

$$\partial_\mu \partial^\mu \psi = 0$$

Modified first-order form:

$$\left\{ \begin{array}{c} \partial_t \psi + \Pi = 0, \\ \partial_t \Pi + \partial^i \Phi_i = 0, \\ \partial_t \Phi_i + \partial_i \Pi = \gamma_2 C_i \end{array} \right.$$

Constraint violations exponentially damped:

$$\partial_t C_i = -\gamma_2 C_i \implies C_i(t) = C_i(0)\, e^{-\gamma_2 t}$$

Constraint Damping: Einstein Equations

With damping terms, evolution equations expanded:

Modified first-order form:

$$
\begin{cases}
\partial_t \psi_{ab} - (1+\gamma_1) N^k \partial_k \psi_{ab} = -N\Pi_{ab} - \gamma_1 N^i \Phi_{iab}, \\
\partial_t \Pi_{ab} - N^k \partial_k \Pi_{ab} + N\psi^{ki}\partial_k \Phi_{iab} - \gamma_1\gamma_2 N^k \partial_k g_{ab} \\
\qquad = 2Ng^{cd}\left(\psi^{ij}\Phi_{ica}\Phi_{jdb} - \Pi_{ca}\Pi_{db} - g^{ef}\Gamma_{ace}\Gamma_{bdf}\right) \\
\qquad - 2N\nabla_{(a}H_{b)} - Nt^c t^d \Pi_{cd}\Pi_{ab} - Nt^c\Pi_{ci}\psi^{ij}\Phi_{jab} \\
\qquad + N\gamma_0\left[2\delta^c_{(a}t_{b)} - g_{ab}t^c\right](H_c + \Gamma_c) - \gamma_1\gamma_2 N^i \Phi_{iab}, \\
\partial_t \Phi_{iab} - N^k \partial_k \Phi_{iab} + N\partial_i \Pi_{ab} - N\gamma_2 \partial_i g_{ab} \\
\qquad = \frac{1}{2} N t^c t^d \Phi_{icd}\Pi_{ab} + N\psi^{jk} t^c \Phi_{ijc}\Phi_{kab} - N\gamma_2 \Phi_{iab}.
\end{cases}
$$

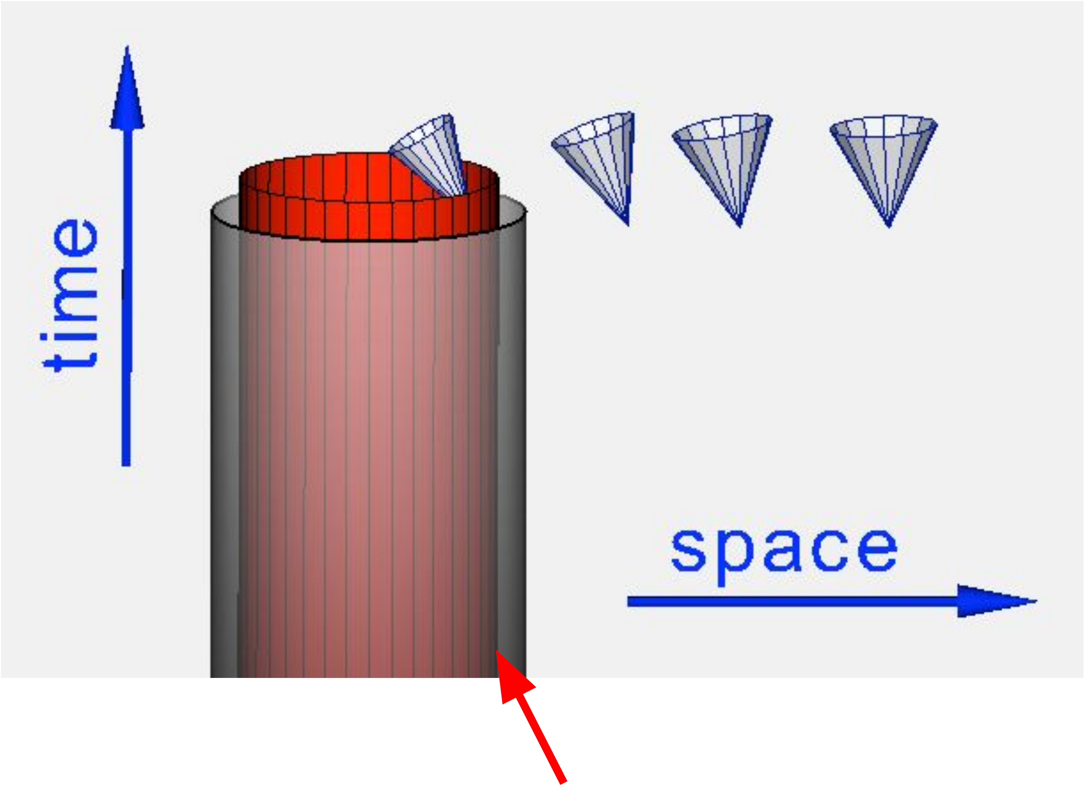
Constraint violations exponentially damped!

$$C_a (= \Gamma_a + H_a) \propto e^{-\gamma_0 t}$$

$$C_{iab} (= \partial_i g_{ab} - \Phi_{iab}) \propto e^{-\gamma_2 t}$$

Lindblom et al, Class.Quant.Grav.23:S447-S462,2006;

Singularity treatment: excision



Excision surface

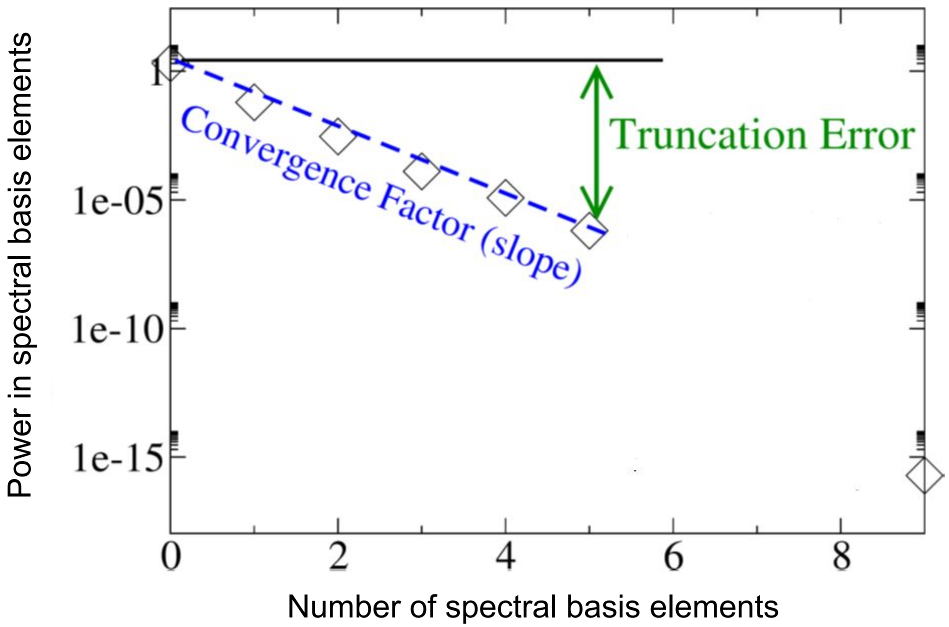
- Formulation of field equations is causal
- No boundary conditions required
- The excision boundary must track the shape and motion of the horizon

Image credit: H. P. Pfeiffer

Robustness: Adaptive Mesh Refinement

Truncation error (or spectral basis representation error) is the primary accuracy diagnostic

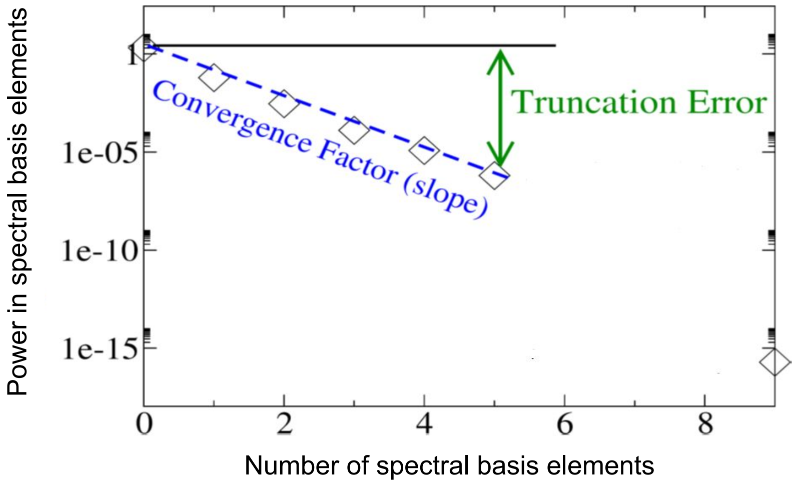
Can be specified and thresholded on in a spacetime dependent manner



Bela Szilagyi, Int. J Mod. Phys. D, Vol. 23, No. 6 (2014) 1430014;

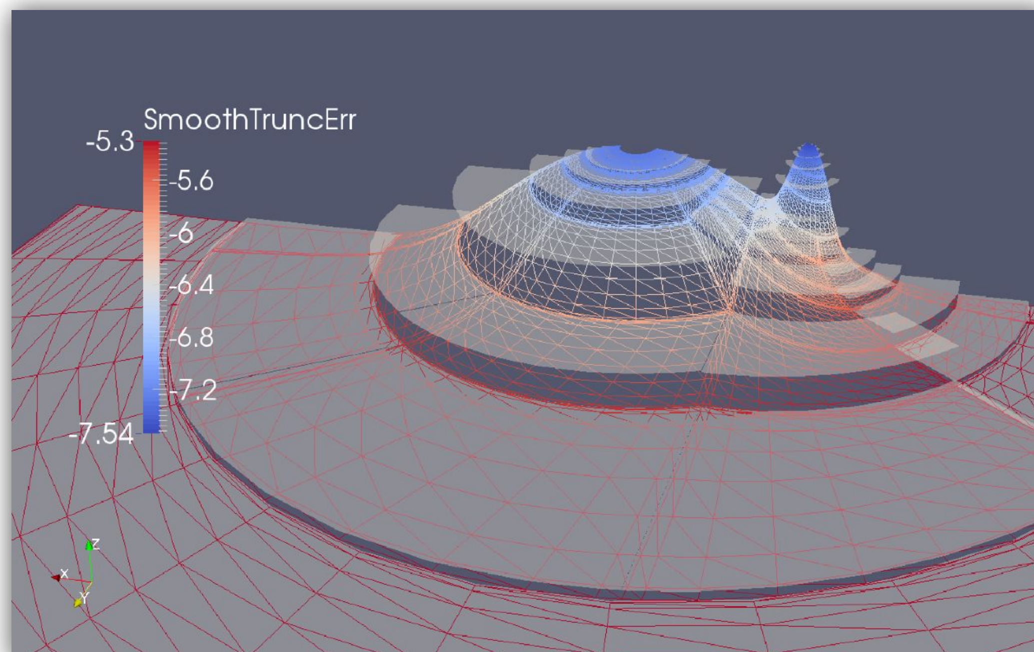
Robustness: Adaptive Mesh Refinement

Truncation error (or spectral basis representation error) is the primary accuracy diagnostic



Can be specified and thresholded on in a spacetime dependent manner

Numerical/Grid resolution is controlled through truncation error. We can get desired resolution in physically more interesting regions, without increasing it in the large wave-zone.

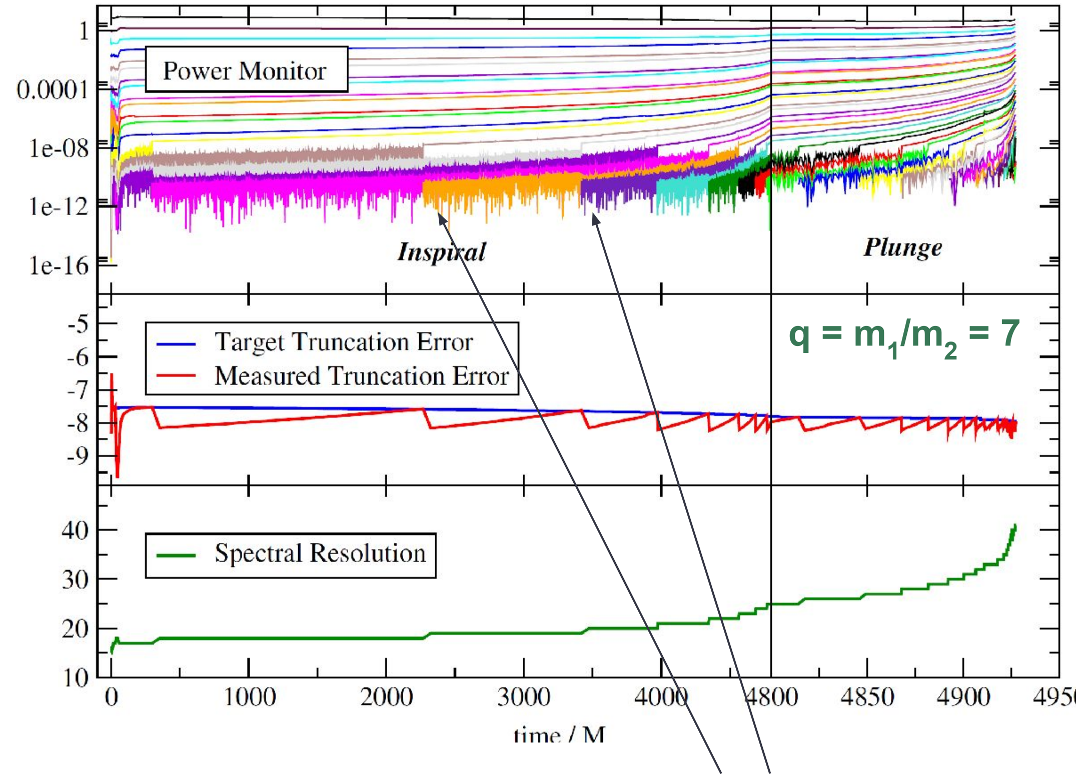


Bela Szilagyi, Int. J Mod. Phys. D, Vol. 23, No. 6 (2014) 1430014;

Robustness: Adaptive Mesh Refinement

Based on truncation error:

Type I: Collocation points added, domain structure unchanged



Add a spectral basis element when:
truncation error > target

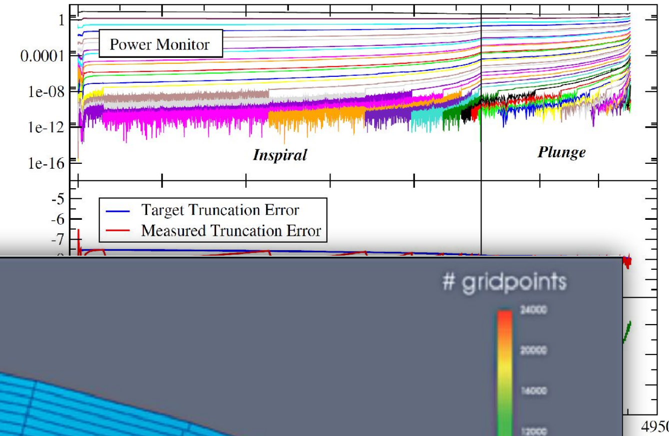
Bela Szilagyi, Int. J Mod. Phys. D, Vol. 23, No. 6 (2014) 1430014;

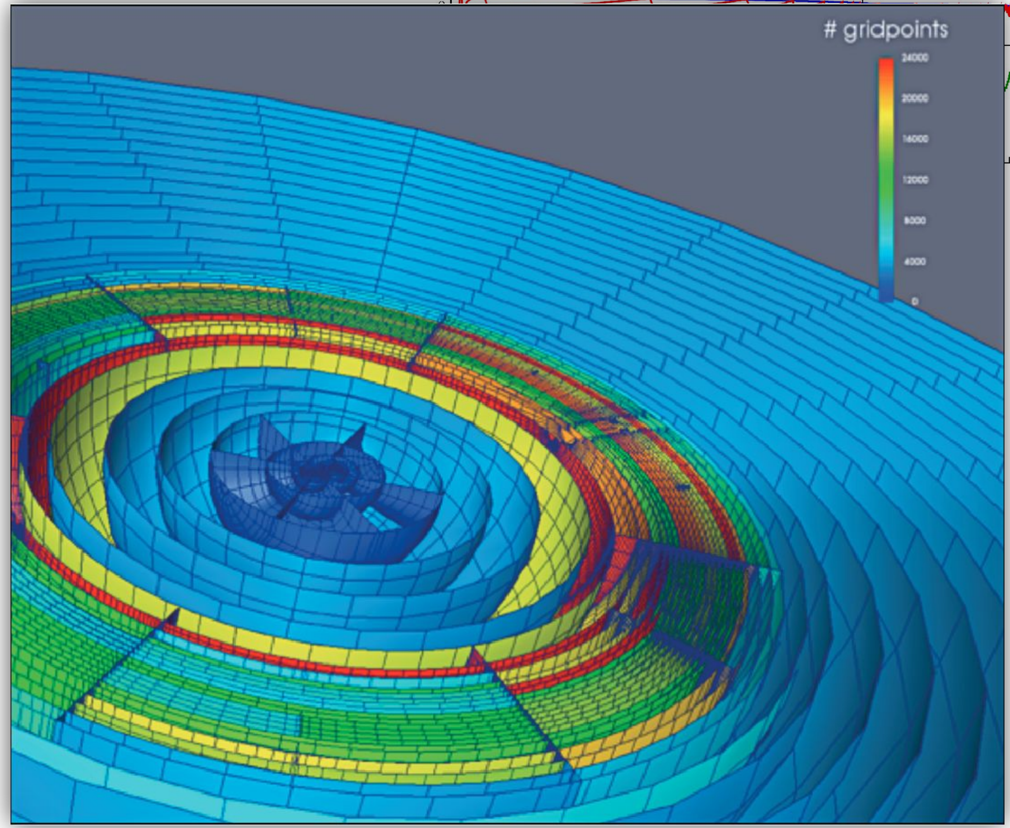
Robustness: Adaptive Mesh Refinement

Based on truncation error:

Type I: Collocation points added, domain structure unchanged

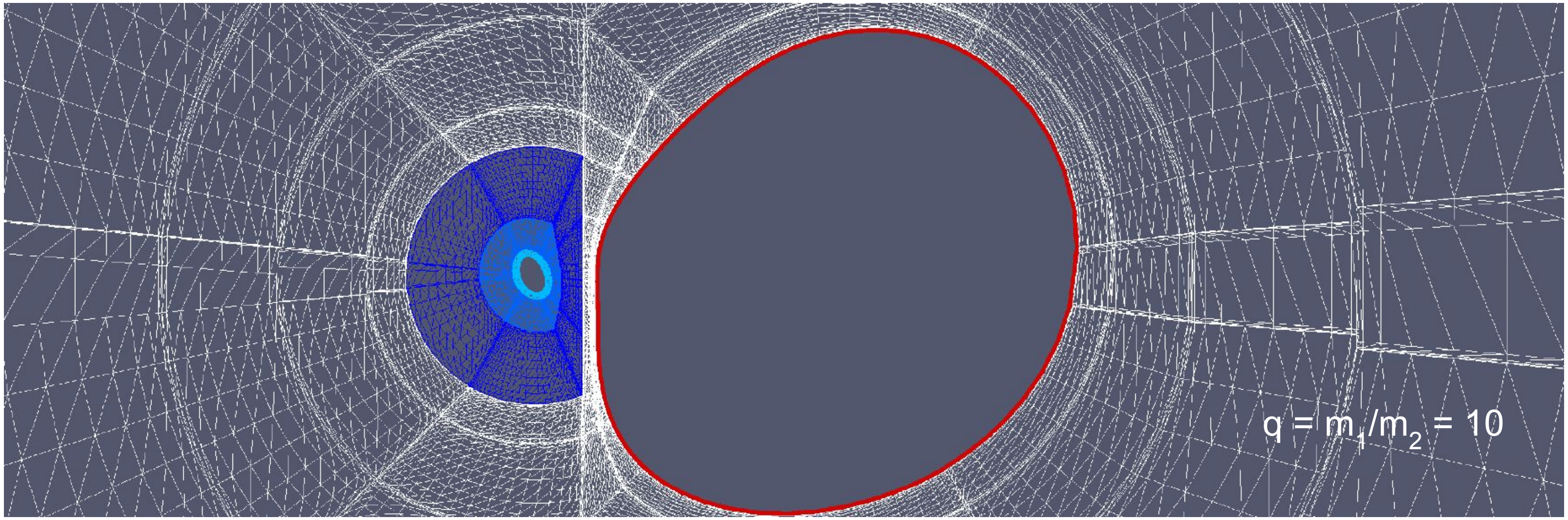
Type II: Sub-domain boundaries re-drawn. Splitting or Merging of subdomains.





Bela Szilagyi, Int. J Mod. Phys. D, Vol. 23, No. 6 (2014) 1430014; Image credit: H. P. Pfeiffer

Robustness: Control Systems



- Compute (apparent) horizons often
- Sub-domains smoothly deformed to track the horizons' shape and position :

$$r_H = \sum_{l,m} R_{lm} Y^{lm}(\theta, \phi)$$

- Feedback-loop control of the coefficients : R_{lm}

Bela Szilagyi, Int. J Mod. Phys. D, Vol. 23, No. 6 (2014) 1430014;

What made it challenging:

Multiple length/time scales, Courant limit, Accuracy required

1. **Multiple length/time scales**
2. **Which coordinates to use (for a spacetime one doesn't know yet)?**
3. **Putting Black holes (singularity) on a grid**
4. **Einstein constraints grew exponentially**
5. **Resolving shocks (discontinuities)**
6. **Computational Challenges**
7. **High accuracy required by LIGO**

What still makes it challenging

1. Multiple length/time scales
2. ~~Which coordinates to use (for a spacetime one doesn't know yet)?~~
3. ~~Putting Black holes (singularity) on a grid~~
4. ~~Einstein constraints grew exponentially~~
5. Resolving shocks (discontinuities)
6. Computational Challenges
7. High accuracy required by LIGO

Spectral Einstein Code (SpEC)

SIMULATING EXTREME SPACETIMES
Black holes, neutron stars, and beyond…

ICTS

What still makes it challenging

1. Multiple length/time scales
2. ~~Which coordinates to use (for a spacetime one doesn't know yet)?~~
3. ~~Putting Black holes (singularity) on a grid~~
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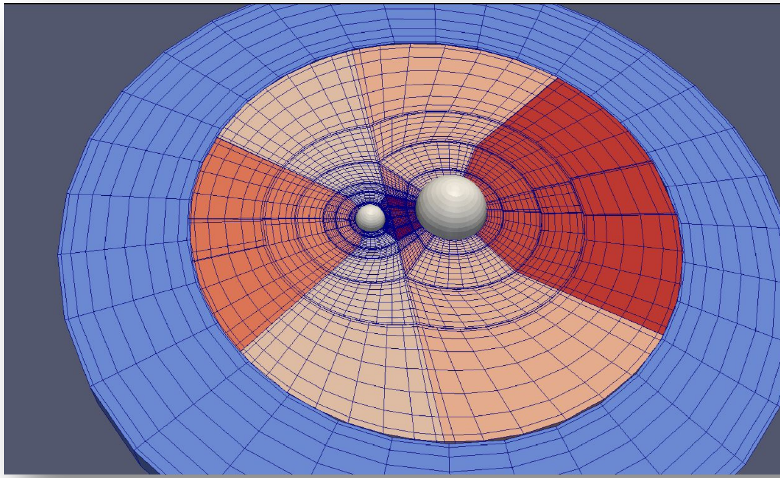
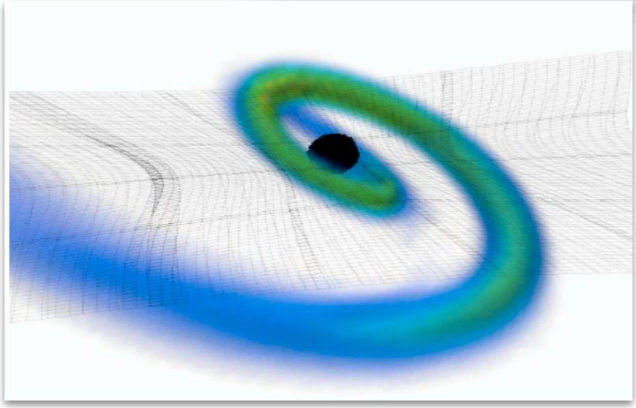
5. Resolving shocks (discontinuities)
6. Computational Challenges
7. High accuracy required by LIGO

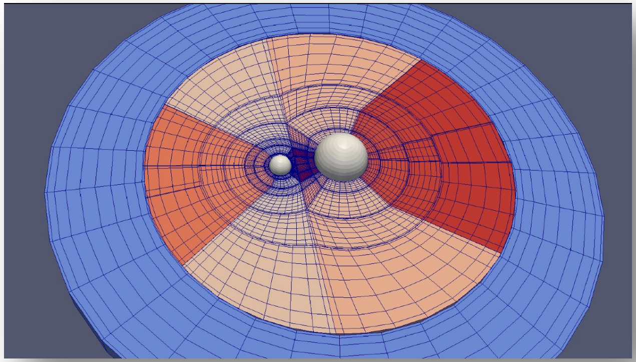




Szilagyi et al (2014); Foucart et al (2016)

Back to the drawing board

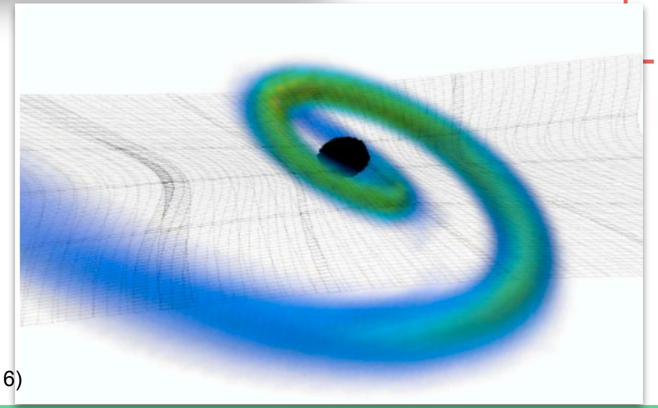
1. Multiple scales
2. Computational Challenges
3. Shocks
4. High accuracy



1. <u>Discretization scheme</u> that:
   a. is local at high order
   b. can handle discontinuities
   c. amenable to inhomogeneous grid

1. <u>Parallelization scheme</u> that can scale, and use all computing available

1. <u>Local time-stepping</u> to handle multiple time scales

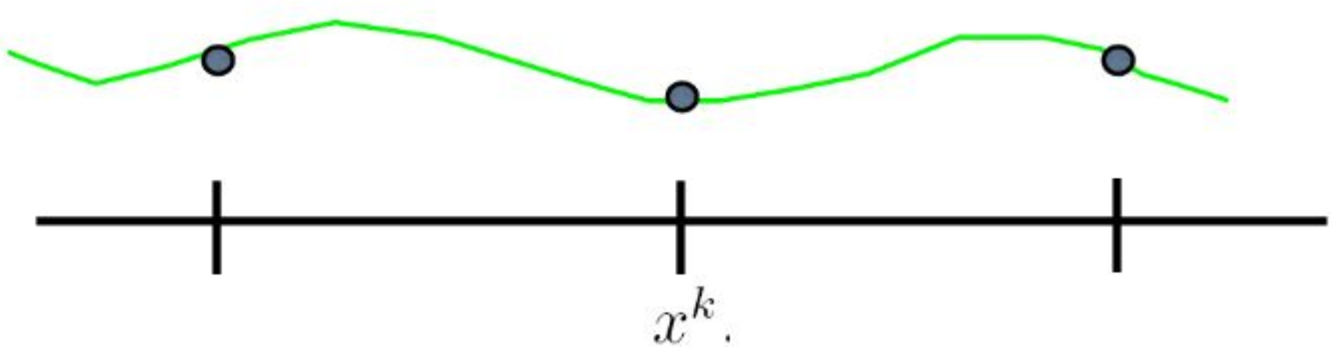


Szilagyi et al (2014); Foucart et al (2016)

Discretization: Finite Difference Methods

- Domain is a **set of collocation points**
- Solution represented **locally as a polynomial**
- Derivatives require stencils

Local at low-order	↑
Local at high-order	↓
Handle discontinuities	↑
Inhomogeneous grids	↑



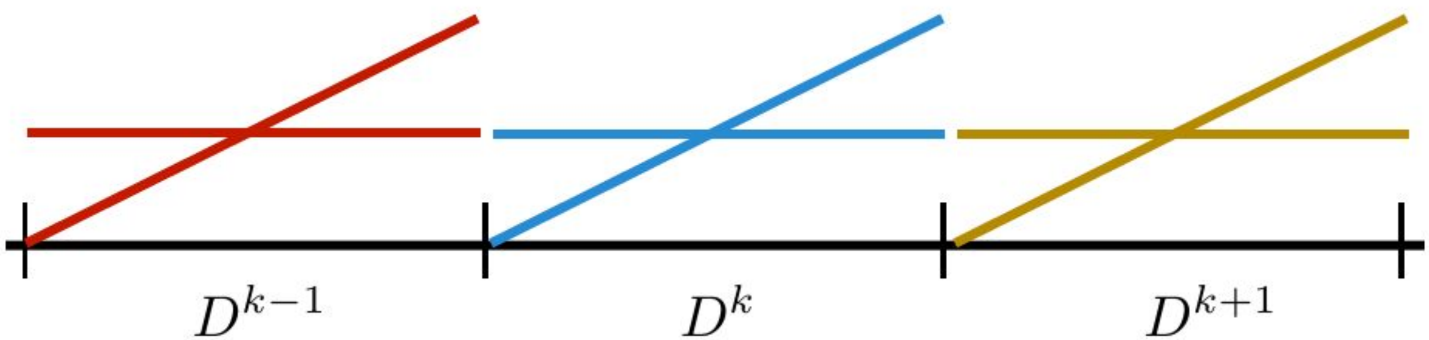
Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Discretization: Spectral Methods

- Solution expanded on a **local basis**

SIMULATING EXTREME SPACETIMES
Black holes, neutron stars, and beyond...

Spectral Einstein Code (SpEC)



Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Discretization: Spectral Methods

- Solution expanded on a **local basis**
- Local high order ⇒ **exponential convergence in smooth regions**

SIMULATING EXTREME SPACETIMES
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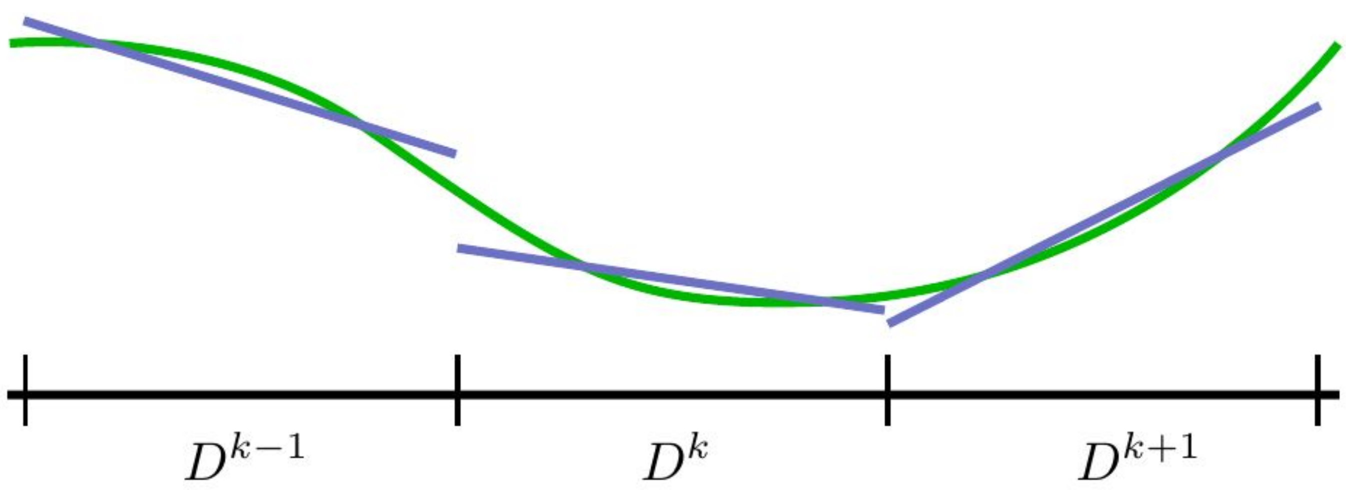
Spectral Einstein Code (SpEC)



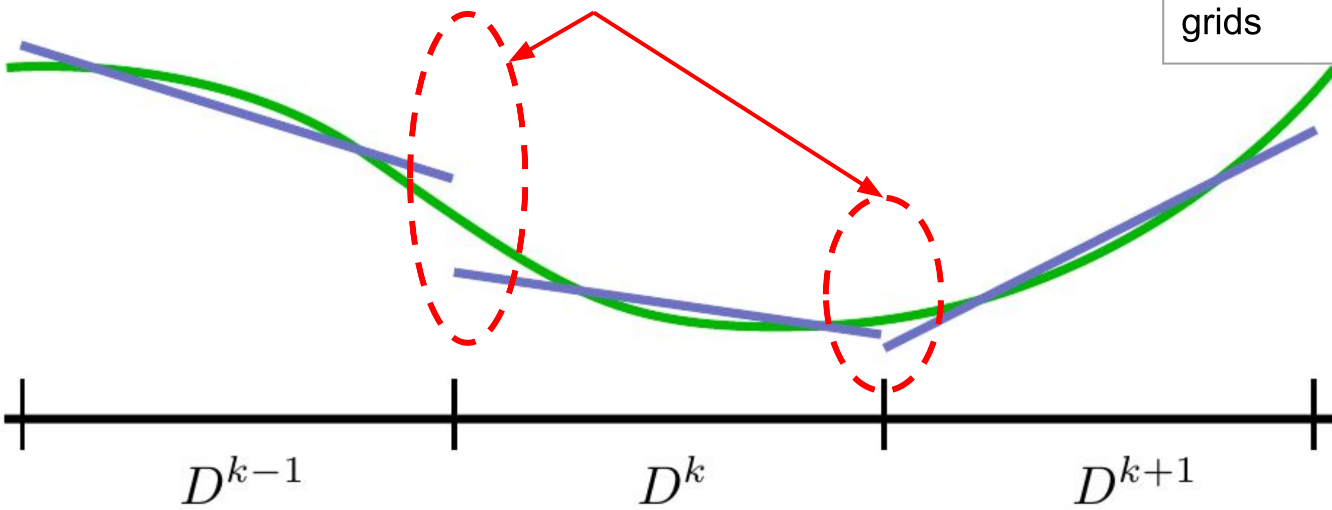
Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Discretization: Spectral Methods

- Solution expanded on a **local basis**
- Local high order ⇒ **exponential convergence in smooth regions**
- … **but flux cannot handle discontinuities / shocks**

Local at low-order	↑
Local at high-order	↑
Handle discontinuities	↓
Inhomogeneous grids	↑

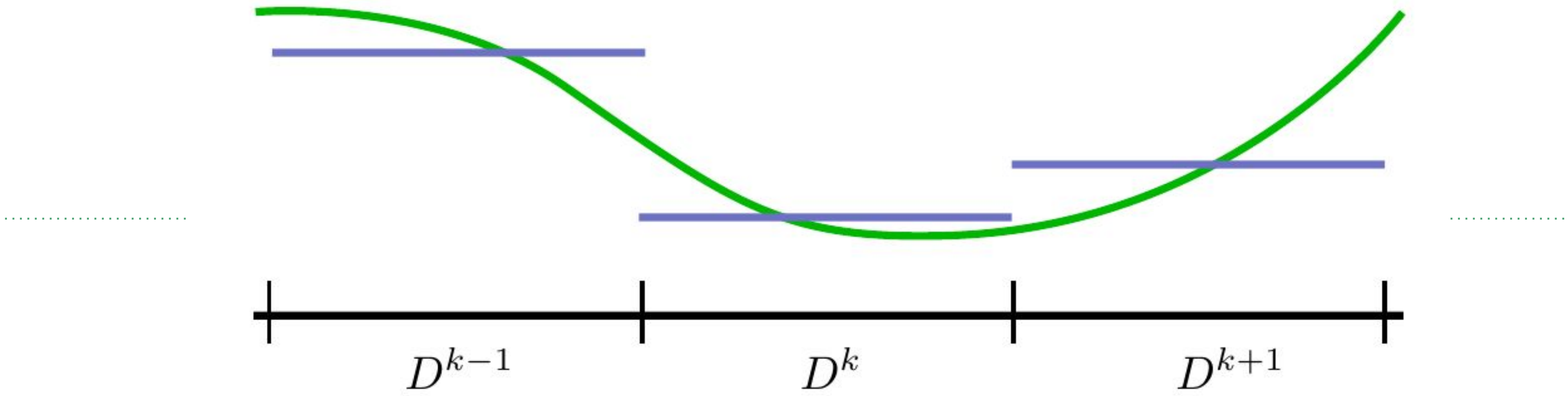
Flux reconstruction requires strict continuity



Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Discretization: Finite Volume Methods

- Solution represented by **cell averages**

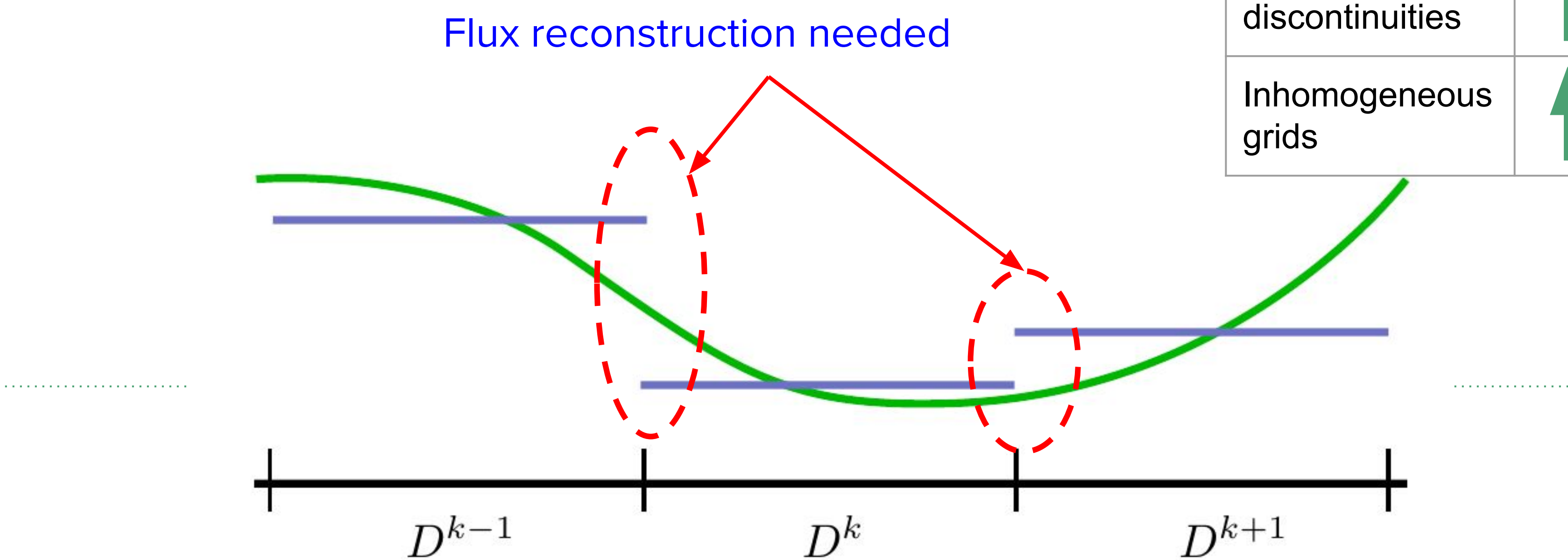


Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Discretization: Finite Volume Methods

- Solution represented by **cell averages**
- Flux reconstruction can handle shocks
- … **but high order requires wide stencils** (as in FD)

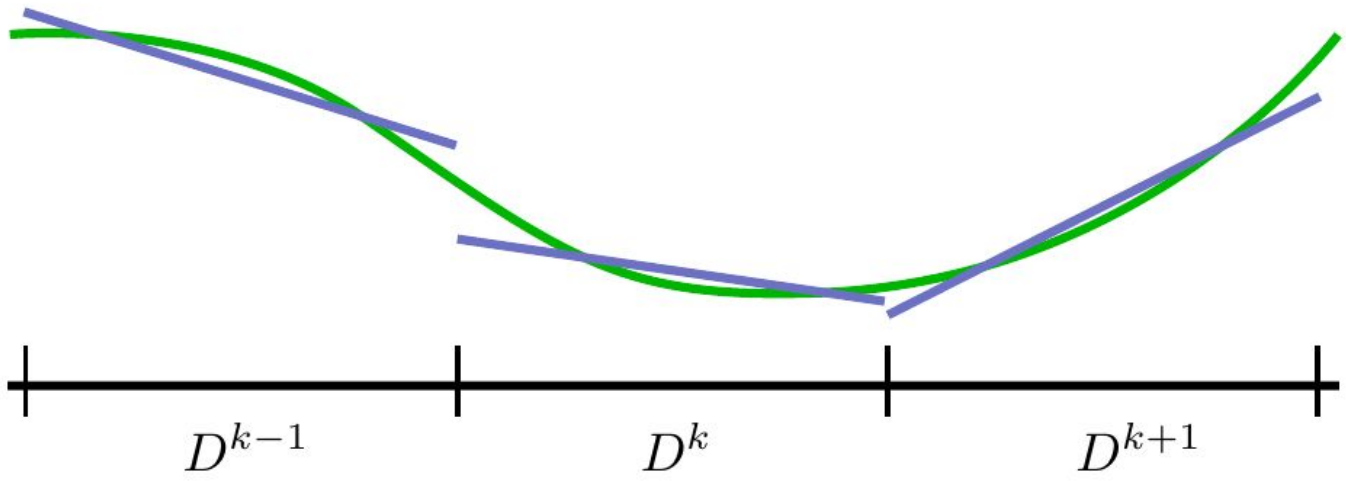
Local at low-order	↑
Local at high-order	↓
Handle discontinuities	↑
Inhomogeneous grids	↑



Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Discretization: Discontinuous Galerkin (DG)

- Solution expanded on a **local basis**

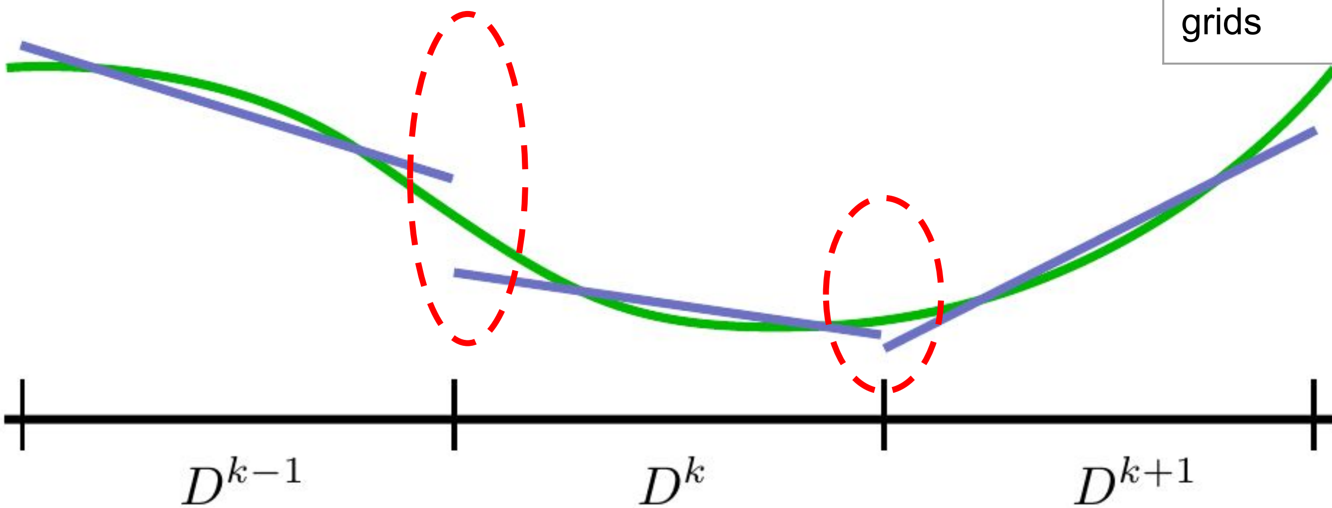


Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Discretization: Discontinuous Galerkin (DG)

- Solution expanded on a **local basis**
- Exponential convergence in smooth regions
- … **and formulation allows "arbitrary" fluxes ⇒ can handle shocks!**

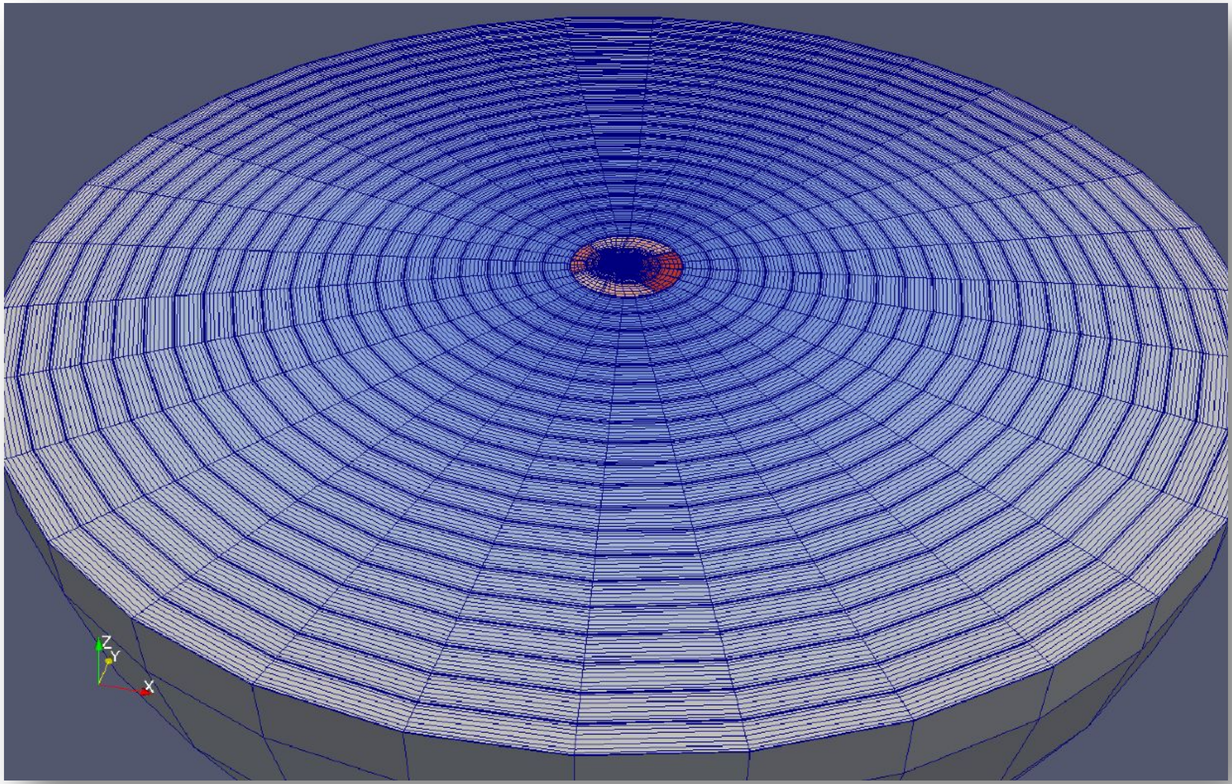
Local at low-order	↑
Local at high-order	↑
Handle discontinuities	↑
Inhomogeneous grids	↑



Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Local time-stepping

- Evolve the solution in time depending on the local needs
- No wastage of computing due to one corner with high-frequency activity



Szilagyi et al (2014)

Local time-stepping

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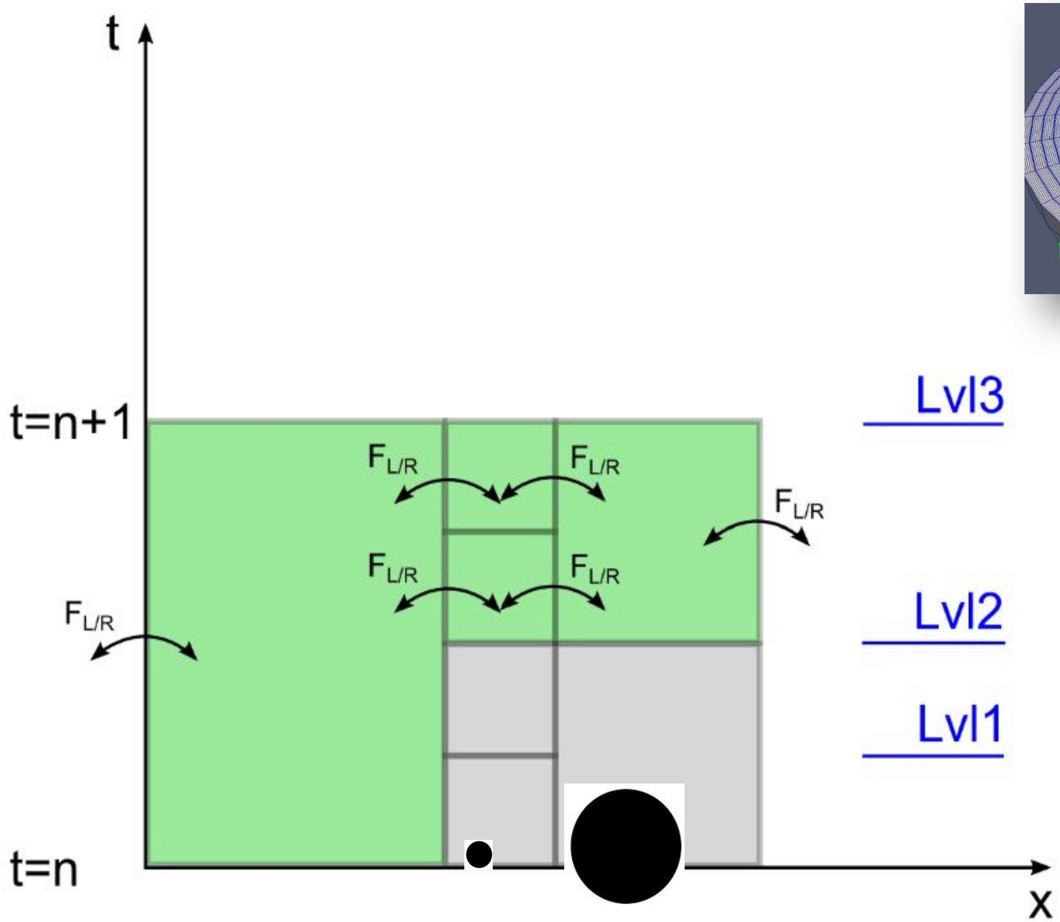
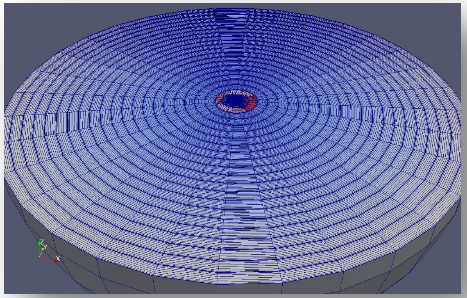
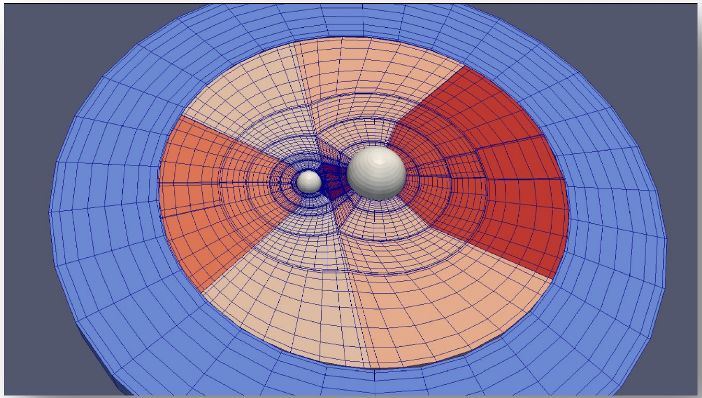


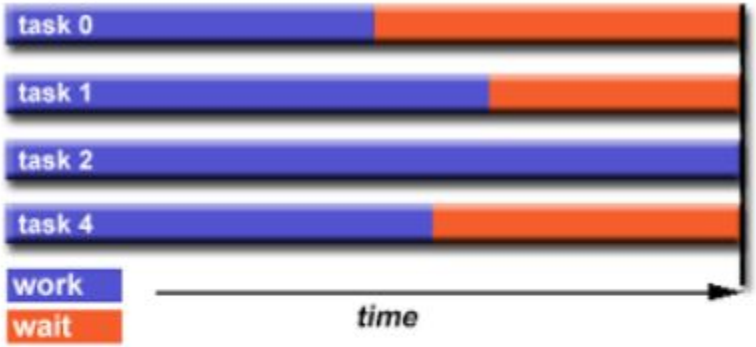


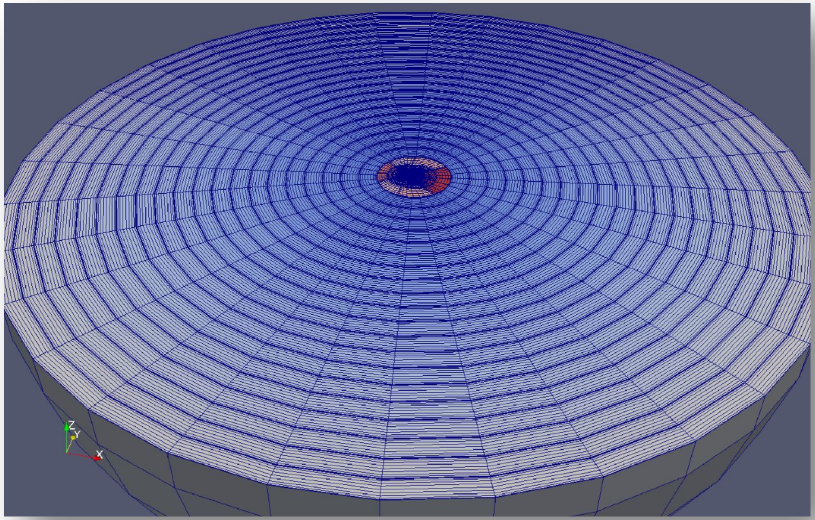
PC: Throwe and Teukolsky

Parallelization scheme: MPI Domain based



- Allocate one domain element per core
- Use MPI



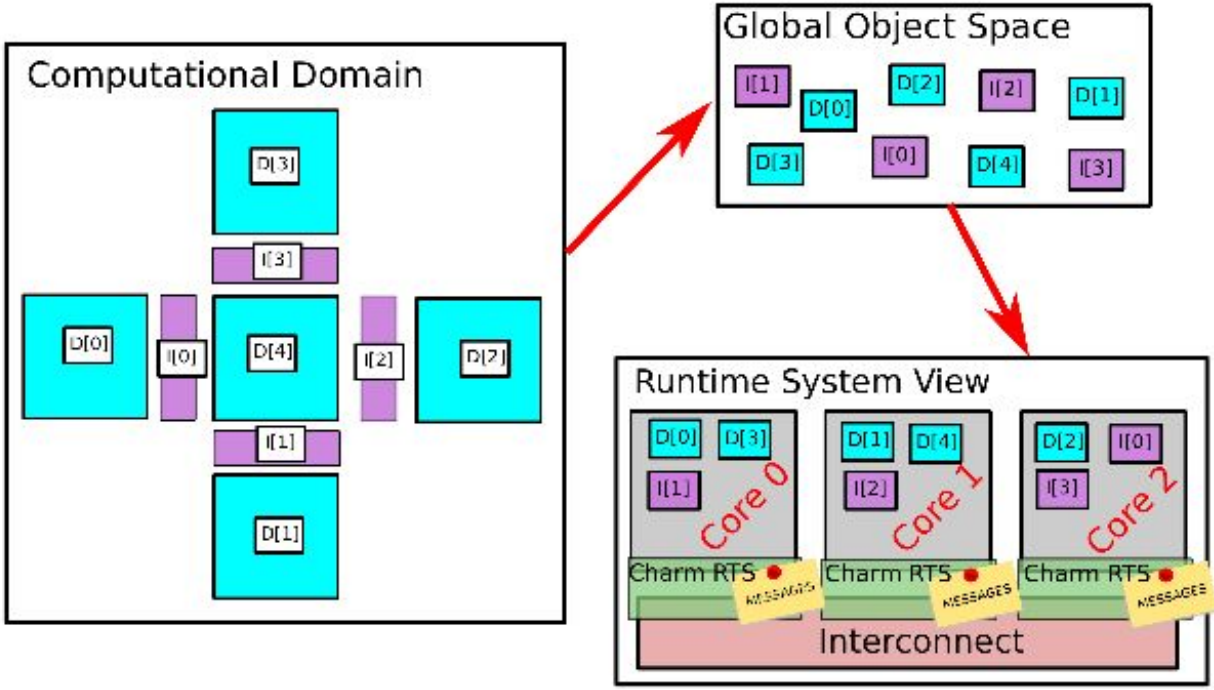


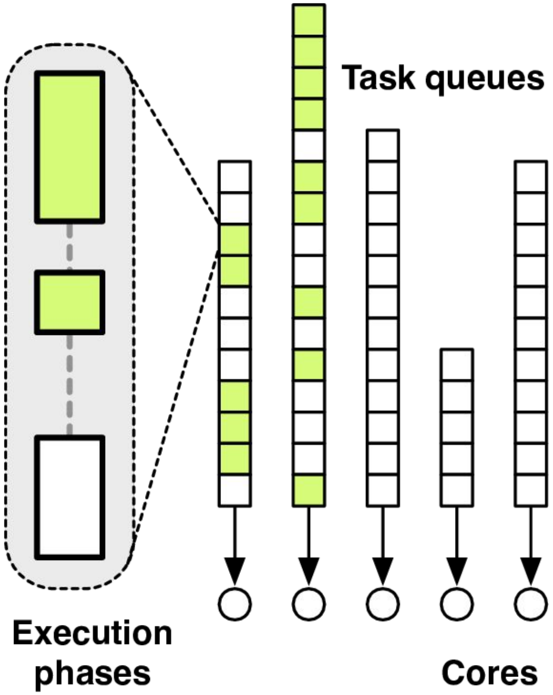
⇒ …terrible terrible idea for systems with length scales that span several orders of magnitude!

Szilagyi et al (2014).

Parallelization scheme: Task-based

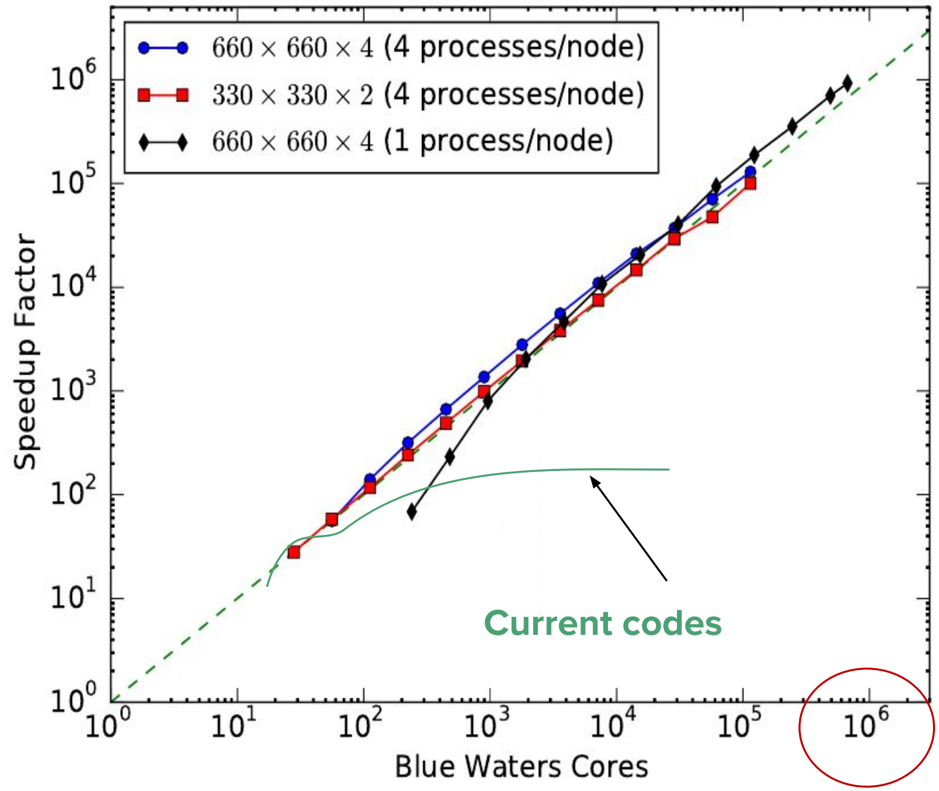
- Divide computation by tasks, not physical domain
- Make communication of data between elements also a task
- Communication-cost hidden behind computation

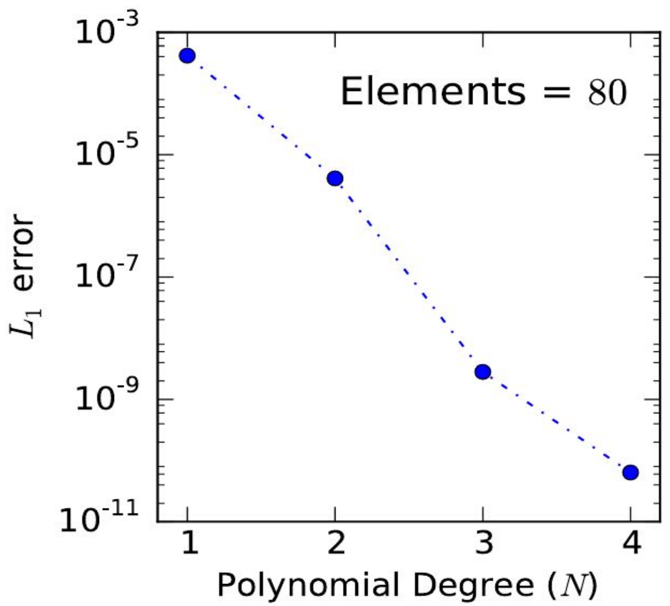




Kidder et al (2016); Stark et al (2014).

SpECTRE: scaling



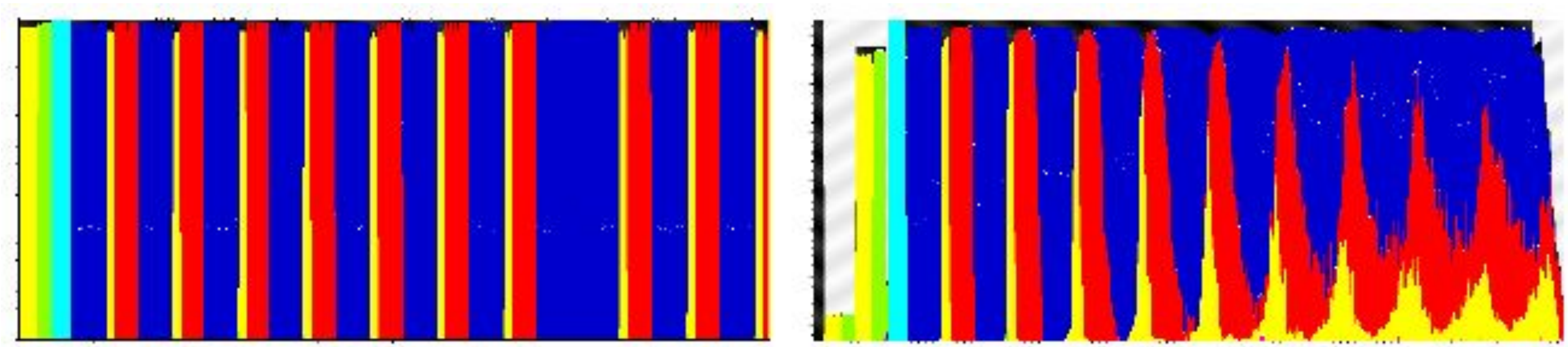


- SpECTRE aims to **combine the high-order accuracy of spectral methods with the local nature of finite-volume/element methods**

- **Future proof:** Computing efficiently scales to ~600, 000 cores. Future proof: exascale computing!

Kidder et al (2016);

SpECTRE: parallelism



(a) 1 core

(b) 12 cores

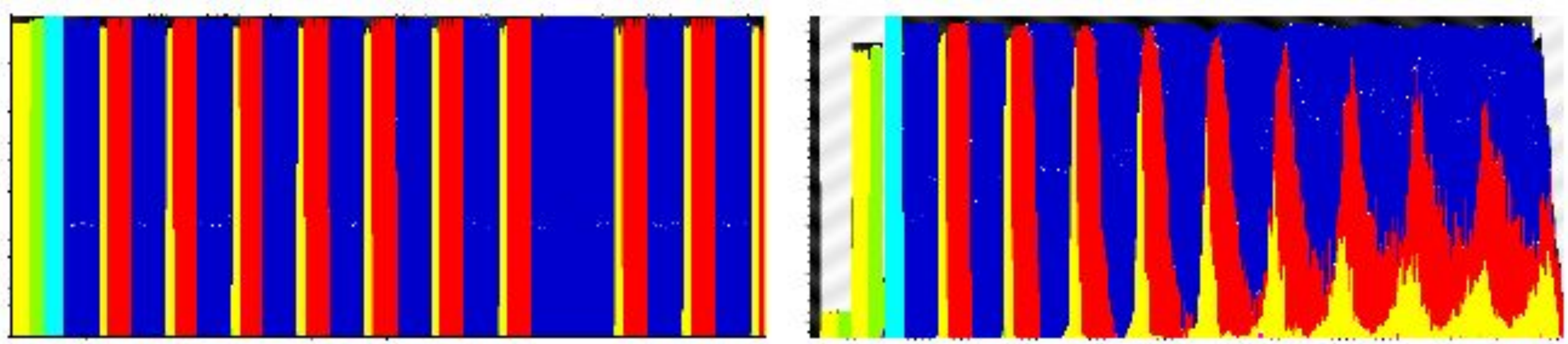
Total CPU Usage

Simulation time

Kidder et al (2016);

SpECTRE: parallelism

Total CPU Usage



(a) 1 core

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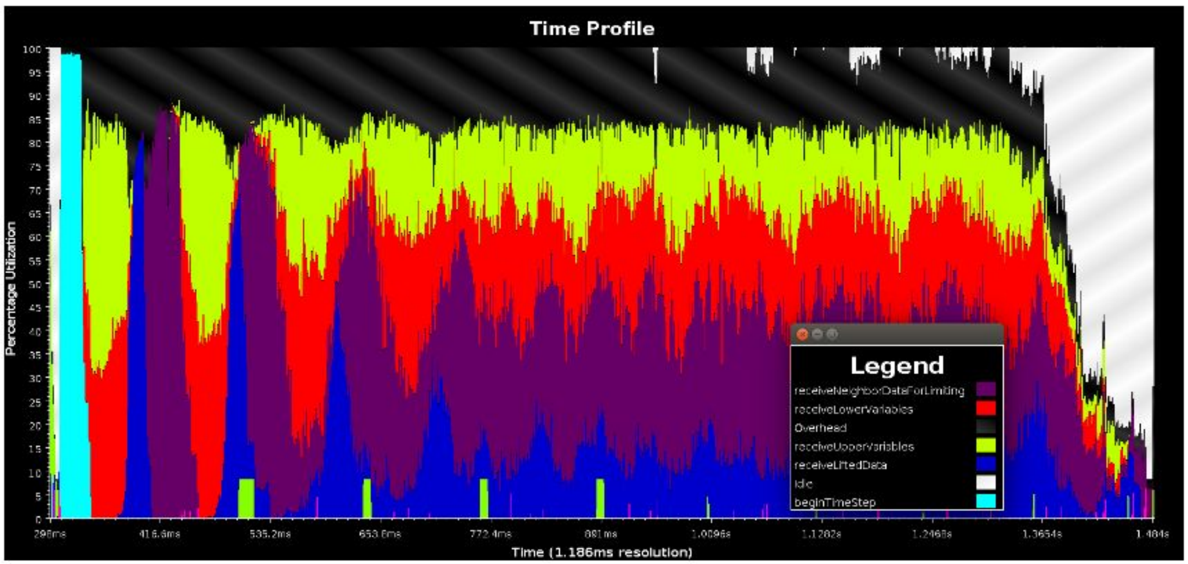
Simulation time



Red/Yellow: data to interfaces (hides RHS vol.)
Blue: fluxes to elements
Cyan: setup

Purple: slope limiting
Black: Charm++

White: idle

Kidder et al (2016);

Summary

1. **Spectre** is a radically forward-looking computational (astro)physics code that adopts cutting-edge computing paradigms:
   a. **DG-FEM discretization**
   b. **Local time-stepping**
   c. **Task-based parallelism**

1. **TBP will enable exascale computing**

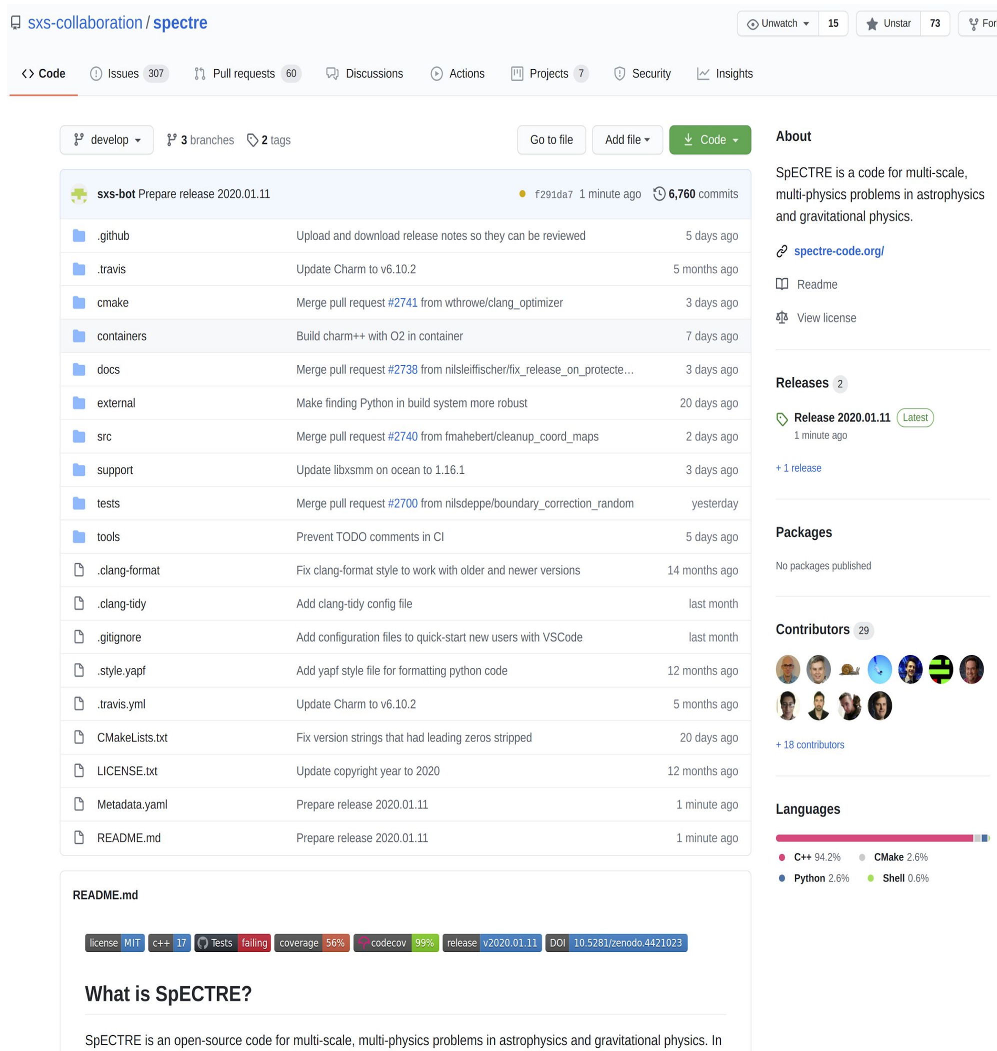
2. Einstein/MHD equations implemented

3. Boundary treatment nearly complete

1. Need control systems!

1. **Spectre is open-source!**

https://github.com/sxs-collaboration/spectre



Thank You for Listening!

Questions?