Isolated and Dynamical Horizons

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Introduction

- Black holes (BH) are objects of immense importance for astrophysics, cosmology and for understanding the nature of gravity itself.
- It is also believed that BH will provide us with a deeper understanding of quantum gravity, just like the hydrogen atom helped us to understand the quantum nature of matter.
- Clasically, BH are a class of solutions of the Einstein (or the appropriate) equations of motion.
- Einstein's Equations: $R_{ab} - \frac{1}{2} R\, g_{ab} = 8\pi G T_{ab}$.
  We shall use $c = 1$ units here.

An example

- An well known example is the Schwarzschild solution.
- Assume: Vacuum ($T_{\mu\nu} = 0$) and Spherical Symmetry.
- $ds^2 = -(1 - \frac{2m}{r})dt^2 + (1 - \frac{2m}{r})^{-1}dr^2 + r^2 d\Omega^2, \quad m = GM.$
- Note the following properties of the metric:

  (a) Metric coefficient diverges at $r = 0$ and at $r = 2m$.

  (b) As $r \to \infty$, the metric becomes Minkowskian.

  (c) The metric has 4 independent Killing Vector Fields (KVF):

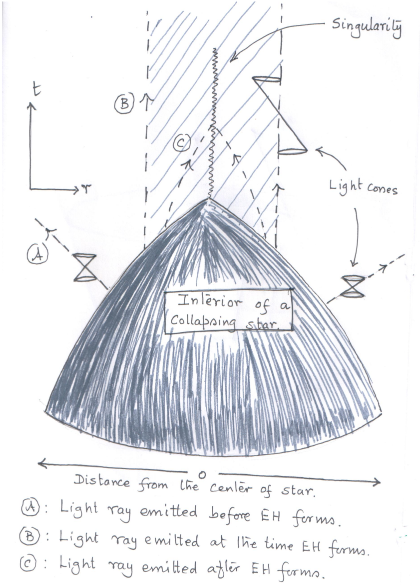
  the time translation generator $t^a := (\partial/\partial t)^a$,

  and the three angular momentum generators L_i^a, $i = 1, 2, 3$.

The spacetime is 4 dimensional of signature $(-+++)$.

- The surface $r=0$ is called the singularity while the surface $r=2m$ is called the Event Horizon (EH).
- Since $t^a t_a = -(1-\frac{2m}{r})$,
  t^a becomes a null/ lightlike vector on the EH.
  At $r\to\infty$, the spacetime is Minkowskian and $t^a t_a = -1$.
  t^a is spacelke for $r<2m$.
- $r=2m$ is a special surface called the Schwarzschild radius.

The formation of EH from gravitational collapse of dust in the homogeneous mass models of Oppenheimer- Snyder (1939) and Dutt (1938).



The EH in marginally bound OSD model

The metric for this spacetime :

$$ds^2 = -dt^2 + R'(r,t)^2\,dr^2 + R(r,t)^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right).$$

Equation for collapsing shell of radius $R(r,t)$, mass $F(r) = m\,r^3$:

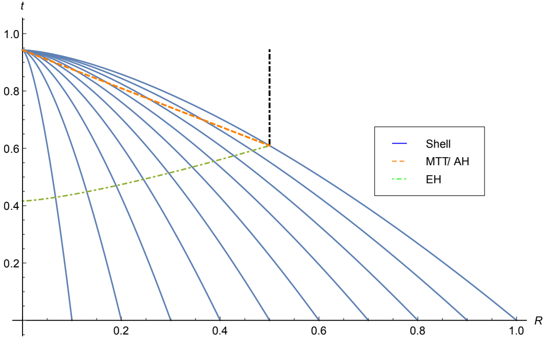
$$R(r,t) = r\left[1 - \frac{3}{2}\frac{\sqrt{F}}{r^{3/2}}\;t\right]^{\frac{2}{3}}.$$

$R(r, t_s) = 0$ is reached by all shells simultaneously at $t_s = (2/3\sqrt{m})$.

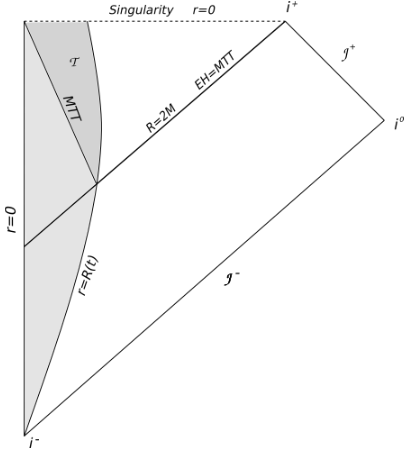
How does the event horizon behave ?

The EH begins as soon as the first shell begin to collapse. This is the teleological nature of the EH:
EH anticipates the formation of a black hole horizon in the future.

The EH is the last null ray reaching the null infinity. Using the matching conditions and mass function, one may track the EH for the OSD as well as LTB (inhomogeneous) collapse models (Chatterjee, Jaryal, Ghosh, PRD 2020).



Unless one has knowledge of the entire future development of the spacetime, EH cannot be located, need the past of null infinity. Determination of EH depends on the final condition rather than the initial condition.



The laws of mechanics for stationary EH

Using stationary BHs (Killing horizons) and their linear perturbations, one obtains laws for transition from one equilibrium state to another nearby equilibrium state.(Bardeen, Carter, Hawking, 1973.).

- Zeroth Law: The *Surface Gravity* (κ) is constant on EH.
- First Law: $\delta M_{ADM} = \frac{\kappa}{8\pi G}\,\delta\mathcal{A}_{\mathcal{H}} + \Omega_H\,\delta J + \cdots$.
- Second Law: $\delta\mathcal{A}_{\mathcal{H}} > 0$. The cross-sectional area of $\mathcal{H}$ can never decrease in a classical dynamical process.

These laws are remarkably similar to the laws of thermodynamics if we identify $(\hbar\kappa/2\pi) \leftrightarrow T$, $M_{ADM} \leftrightarrow E$ and $(\mathcal{A}_{\mathcal{H}}/4G\hbar) \leftrightarrow S$.

Bekenstein (1973), Hawking (1973,1974).

Thermodynamics for EH

- It is natural to identify black holes as thermal objects, if quantum effects are also included (Hawking, 1974).

- In String Theory as well as in Loop Quantum Gravity, these ideas have led to important developments including counting of microscopic states leading to black hole entropy:

$$S = \frac{\mathcal{A}}{4l_P^2} + \alpha \ln \frac{\mathcal{A}}{4l_P^2} + \beta \frac{4l_P^2}{\mathcal{A}} + \cdots + \exp(-\delta \frac{\mathcal{A}}{4l_P^2}),$$

where α, β are constants, $l_P^2 = G\hbar$.

(Strominger, Vafa; Sen; Ashtekar *et al*; Smolin, Rovelli, Kaul-Majumdar; Carlip; Ghosh- Mitra; Perez, $\cdots$).

Quick recap

- EH are not a good description of black hole horizon.
  Need the knowledge of the entire future of the spacetime.
  However, this has led to important discoveries.

- The task is to use a description of horizon which is (quasi)- local, but may be useful to understand the classical and quantum mechanical properties of black holes.

Aim of the talk

Use a better formulation of horizons.

- Explain the notion of local horizons: Marginally trapped tube (MTT), Isolated (IH) and Dynamical horizons (DH).

  (Ashtekar, Galloway, Lewandowski, Bettle, Krishnan, Fairhurst, Krasnov, Booth, . . .)

- Show how these horizons change in classical and quantum processes: Laws of black hole mechanics, and flux laws on horizons.

  (Ashtekar- Krishnan; Chatterjee, Ghosh, Jaryal, Kumar, . . .)

- Local Lorentz charges on horizon: BH entropy.

  (Chatterjee, Ghosh, Devdutt)

Our Group and Collaborators

This talk is based on the work with the following researchers.

- Gravity Group at CUHP

  Ayan Chatterjee
  Akshay Kumar
  Sahil DevDutt
  Akriti Garg.

- Collaborators.

  Amit Ghosh (Saha Institute of Nuclear Physics)
  Avirup Ghosh (Kulti College)
  Suresh Jaryal (Jammu Central University)

References for this talk

The references for this talk are:

Jaryal, Chatterjee, Kumar, EPJC, (2024)

Jaryal, Chatterjee, Phy. Dark Uni (2022), EPJC (2021)

Chatterjee, Ghosh, Jaryal, PRD (2022)

Chatterjee, Ghosh PRL (2020)

Chatterjee, Ghosh PRD (2021)

Chatterjee, Ghosh, Jaryal PRD (2020).

Trapped Surfaces

The formulation of trapped surface by Penrose is useful in this respect (Penrose, 1965).

Definition: A 2- dimensional closed spacelike submanifold $\mathcal{S}$ of a 4-dimensional spacetime $\mathcal{M}$ is called trapped if both the null normals to $\mathcal{S}$ have negative expansion, $\theta_{(\ell)} < 0$ and $\theta_{(n)} < 0$

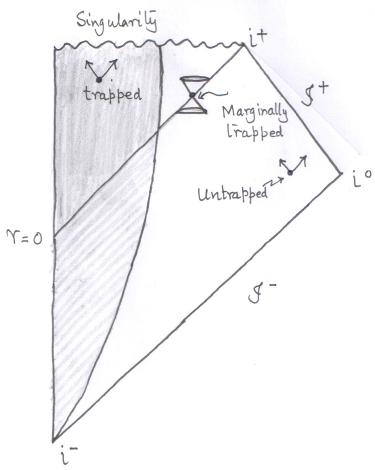
- The trick is to draw a surface $\mathcal{S}$ and see how do the null normals ℓ^a and n^a behave in the future.

  If the area bounded by infinitesimally separated null normals increase, we call expansion to be positive, otherwise negative.

All $\mathcal{S}$ are trapped in BH region and marginally trapped on the boundary.

In the Minkowski spacetime, all $\mathcal{S}$ are untrapped.

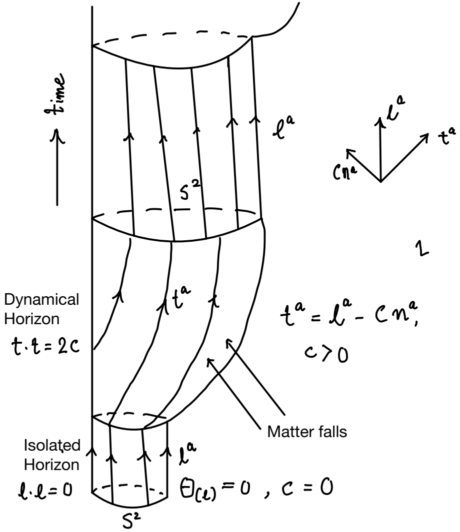
Trapped surfaces capture the essential feature of a black hole region: outgoing light rays cannot escape to observers at infinity.



The boundary of a black hole region is a 3- dimensional submanifold such that each spacelike 2 dimensional cross- section $\mathcal{S}$ is marginally trapped: $\theta_{(\ell)} = 0$ and $\theta_{(n)} < 0$.

It is natural to identify this boundary with the black hole horizon.

- Unlike the EH which is always null, this horizon acquires different signatures.
- It is null when no matter /radiation falls on it, spacelike when matter/radiation falls through it and timelike in other situations.


time
ℓ^a
ℓ^a
t^a
Cn^a
S^2
Dynamical Horizon
$t \cdot t = 2C$
t^a
$t^a = \ell^a - Cn^a$,
$C > 0$
Matter falls
Isolated Horizon
ℓ^a
$\ell \cdot \ell = 0$
$\Theta_{(\ell)} = 0$, $C = 0$
S^2

MTT as a Black Hole horizon

Definition: A 3- dimensional submanifold Δ in a 4- dimensional spacetime is called Marginally trapped tube (MTT) if:
(Ashtekar and Galloway, 2005)

- Δ is topologically $\mathcal{S} \times \mathbb{R}$.
- Each $\mathcal{S}$ is marginally trapped: $\theta_{(\ell)} = 0$ and $\theta_{(n)} < 0$.

If t^a is tangential to Δ and normal to foliations, $t^a = \ell^a - Cn^a$.

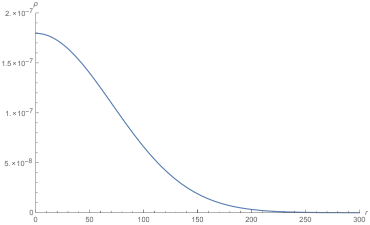
We call t^a to be the generator of Δ.

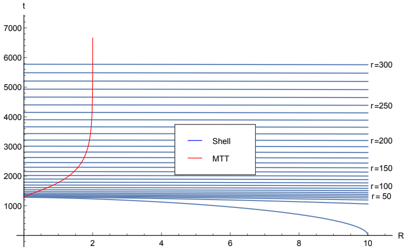
Since $t \cdot t = 2C$, the constant C determines the signature and stability of Δ (Andersson, Mars, Simon, 2008).

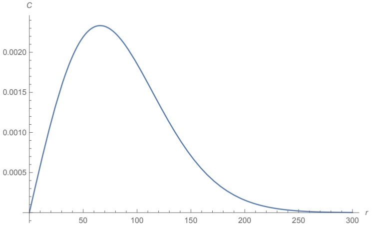
$C ><= 0$ for spacelike, timelike or null.

MTT is a good quasilocal description of BH boundary/horizon. Unlike EH which is always null, this horizon can be spacelike/ null/ timelike.

Gravitational collapse of a pressureless Gaussian profile

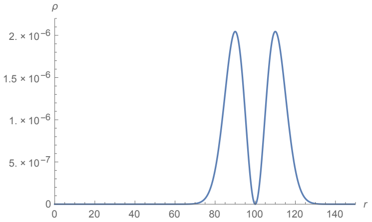




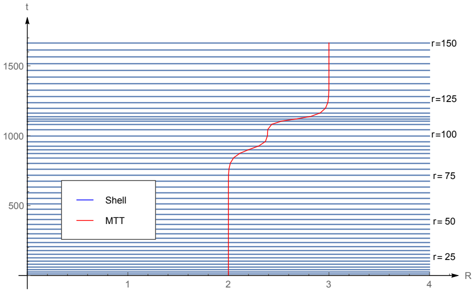


Chatterjee, Jaryal and Ghosh, 2022

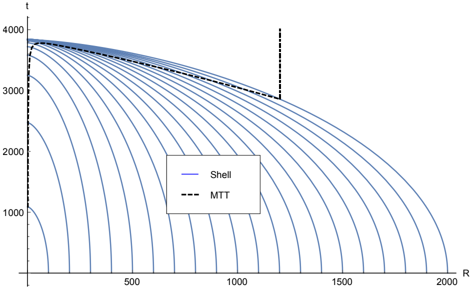
Two shells falling one after the other on a black hole.



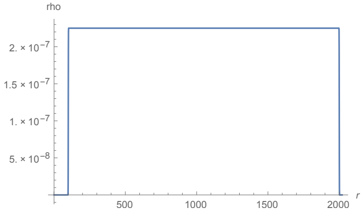
a: Mass profile

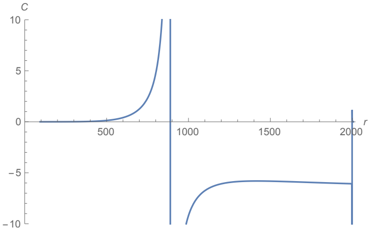


Large mass falling on a black hole.

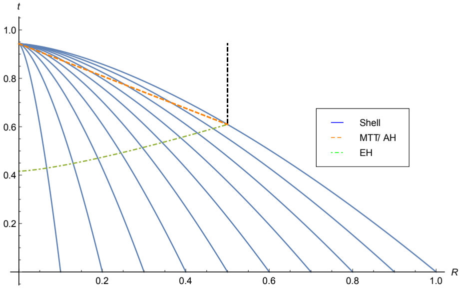


a: MTT for Gaussian mass profile

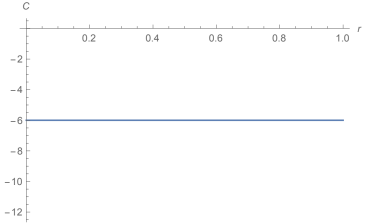




The OSD model.



a: The MTT is timelike and unstable



Isolated Horizons

When no matter falls in, Δ is in equilibrium, $C = 0$ and ℓ^a generates Δ:

- ℓ^a is a Killing vector field on Δ.
- The horizon area does not change: $\mathcal{L}_\ell\, {}^2\epsilon \triangleq 0$.
- The (local) zeroth and the (local) first law of black hole mechanics follows (Ashtekar *et. al.* 2000; Chatterjee and Ghosh, 2009).
- $\kappa_{(\ell)} =$ constant on Δ.
- $\delta M_\Delta = \frac{\kappa_{(\ell)}}{8\pi G}\,\delta\mathcal{A} + \Omega\,\delta J \cdots$, on Δ.

IH: local Lorentz charges on Δ

- GR is invariant under both diffeomorphisms as well as local Lorentz transformations.

- Do the local Lorentz transformations generate charges in the presence of horizons?

- Isolated horizon Δ is characterised by the equations of MTT, and only those transformations are allowed which transform these conditions homogeneously.

- Naturally, in the bulk, the full $SL(2, C)$ is allowed.

- On the Horizon (Δ) the only allowed Lorentz transformation belong to $ISO(2) \ltimes R$ (Basu, Chatterjee and Ghosh, 2012).

- This is not surprising since $ISO(2) \ltimes R$ is the little group of the Lorentz group which keeps the lightlike vector ℓ invariant (Weinberg, QFT).

- The four infinitesimal generators of $ISO(2) \ltimes R$ may be written in the $\ell_I, n_I, m_I, \bar{m}_I$ basis.

- $R_{IJ} = 2im_{[I}\bar{m}_{J]}$, generates rotations in $(m - \bar{m})$ plane.

- $P_{IJ} = 2m_{[I}\ell_{J]} + 2\bar{m}_{[I}\ell_{J]}$ and $Q_{IJ} = 2im_{[I}\ell_{J]} - 2i\bar{m}_{[I}\ell_{J]}$ generate rotations in the $(\ell - m)$ and $(\ell - \bar{m})$ planes respectively.

- $B_{IJ} = -2\ell_{[I}n_{J]}$ generates boosts (scaling of null vectors) in the $(\ell - n)$ plane.

- It is possible to determine the Hamiltonian charges which generate these transformations on Δ.

  See Chatterjee and Ghosh (2018), Chatterjee and Ghosh, (2020).

- To determine the charges for these generators, we use the Holst action. This action is classically equivalent to the Einstein- Hilbert action.
- If we denote the Hamiltonian for R_{IJ} as $-J$, then it can be shown that $(\mathcal{A}/8\pi G\gamma) = J$.
- If K is the Hamiltonian charge for B_{IJ}, it arises that $(\mathcal{A}/8\pi G) = K$.
- The phase space charges generated for P_{IJ} and Q_{IJ} are called Q and P respectively. These charges vanish identically on Δ.
- So, we get two classical expressions: (1) $\mathcal{A} = 8\pi G\gamma J$, and (2) the simplicity constraint $K = \gamma J$.
- γ is the Barbero- Immirzi parameter.

- These quantum algebra of these charges also satisfy the $ISO(2)$ algebra:

  $$[J, P] = i\hbar Q, \;\; [J, Q] = i\hbar P, \quad [P, Q] = 0.$$

- So, in the quantum theory, the states on the cross- sections of Δ must be in the representation of $ISO(2)$.

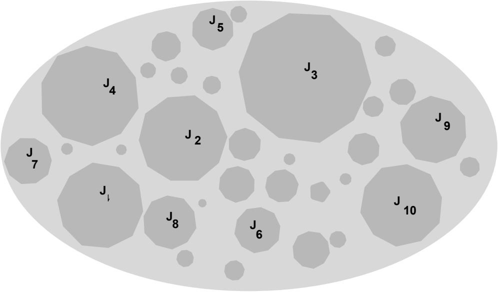
- These states are labelled by a single quantum number $|j\rangle$ ($j \in \mathcal{N}$). These are also the eigenstates of J.

- The spectrum of the area operator is then $\mathcal{A}|j\rangle = 8\pi G\gamma J\,|j\rangle = 8\pi G\hbar\gamma j\,|j\rangle$.

- Area spectrum is equally spaced.

  (Bekenstein- Mukhanov 1990s,
  Alekseev- Polychronachos- Smedback, 2002).

- In the quantum theory, S_Δ is assumed to be tessellated by number of patches, much like the surface of a soccer ball.



- The quantum state of area $\mathcal{S}$ is labeled by an integer $|J\rangle = \otimes_i |j_i\rangle$ where i is the label for the patches.

- The area operator acts on the tessellated surface as $\mathcal{A} = \oplus_i \mathcal{A}_i$ where each of the area patch contributes an area $8\pi\gamma\ell_p^2 j_i$, such that $J = \sum_i j_i$.

- The entropy is obtained by determining the number of independent ways the configurations $\{j_i\}$ can be chosen such that for a fixed J the condition $J = \sum_i j_i$ is satisfied.

- The entropy is given by

$$S = \frac{\mathcal{A} \ln 2}{8\pi\gamma l_p^2} + \alpha e^{-\mathcal{A} \ln 2/8\pi\gamma l_p^2},$$

  and for the choice $\gamma = \ln 2/2\pi$, the leading order Bekenstein-Hawking result is obtained, but also gives an exponentially suppressed correction to the *classical* result. Should become dominant for small areas. These results are generic and independent of Loop Quantum Gravity/ String theory.

- The result is valid even for l_P size black holes, where the exponential corrections become important.

Hamiltonian of boost is the Wald entropy: Lovelock gravity

- The expression of horizon area as a Hamiltonian charge for boost extends to Lovelock gravity as well (ongoing work with DevDutt).
- Action for Lovelock theory on a D dimensional manifold M is:

$$S_L = \int_M \sum_{p=0}^{[D/2]} \alpha_p L^{(p)}$$

$$L^{(p)} = \epsilon_{a_1 \dots a_{2p} a_{2p+1} \dots a_D} F^{a_1 a_2} \wedge \dots \wedge F^{a_{2p-1} a_{2p}} \wedge e^{a_{2p+1}} \wedge \dots \wedge e^{a_D}$$

- $D = 4, \qquad \Omega(\delta_\lambda, \delta) = -\delta \mathcal{A}_\Delta$
- $D = 5 \qquad \Omega(\delta_\lambda, \delta) = \delta \oint_{S_\Delta} {}^3\epsilon \, (1 + 2\alpha \mathcal{R})$
- $D = 7$

  $\Omega(\delta_\lambda, \delta) = -\delta \oint_{S_\Delta} {}^5\epsilon[1 + 2\alpha_2 \mathcal{R} + 3\alpha_3(\mathcal{R}^2 - 4\mathcal{R}^{ab}\mathcal{R}_{ab} + \mathcal{R}^{abcd}\mathcal{R}_{abcd})]$

Summary of the second part

- The formulation of MTT as a horizon is useful for classical and quantum black holes.

- The laws of black hole mechanics holds on the IH.

- Wald entropy is the hamiltonian charge for the boost transformation on IH.

- The horizon area must be discrete with an equally spaced area spectrum, $\mathcal{A} = 8\pi G\hbar\gamma j$ with $j \in \mathcal{N}$.

- Further, the isolated horizon has an the leading entropy given by the Bekenstein- Hawking value:

$$S = \frac{\mathcal{A}}{4l_p^2} + \alpha e^{-\mathcal{A}/4l_p^2},$$

Dynamical horizons: Supertranslations

- The section of the MTT for which $C \neq 0$, is the dynamical horizon (DH).
- Super translation symmetries exist on a IH, but is it a symmetry on a Dynamical horizon ? No.
- Induce supertranslations on a future IH through a flux of radiation through a DH.
- On IH, the super translation may be used to choose good foliations: $\nabla_A \omega^A = 0$.
- So, is it possible to induce $\nabla_A \omega^A \neq 0$ through a flux? Yes, This is like the Bondi news.
- One may show that such terms are induced through a physical process where T_{ab} violates energy conditions.

Hawking radiation

- Consider a spherically symmetric background metric

  $$ds^2 = -2e^{-f}dx^+dx^- + R^2(d\theta^2 + \sin^2\theta d\phi^2)\ .$$

- For the Hawking radiation, the generator of DH, t^a is timelike with $C < 0$.

- This implies that $\mathcal{L}_t\mathcal{A} < 0$, area of horizon decreases.

- How much is the rate of decrease of area? This is the flux law:

$$\mathcal{F} = \int d\mu\ T_{\mu\nu}\hat{n}^\mu K^\nu = -\frac{1}{2}(R_2 - R_1).$$

Open questions and future directions

- Is MTT the true boundary of the black hole region ? Eardley's conjecture: The outer boundary of the region containing outer trapped surfaces is the event horizon.
- What is the role of non- spherical trapped surfaces ? They may extend out to the flat region of the spacetime.
- Need a complete understanding of the corrections to entropy.
- Is there a MTT based derivation of the Hawking radiation beyond spherical symmetry? Need to go beyond the Isolated framework.

Thank You